dK-series:
Systematic Topology Analysis and Generation Using Degree Correlations

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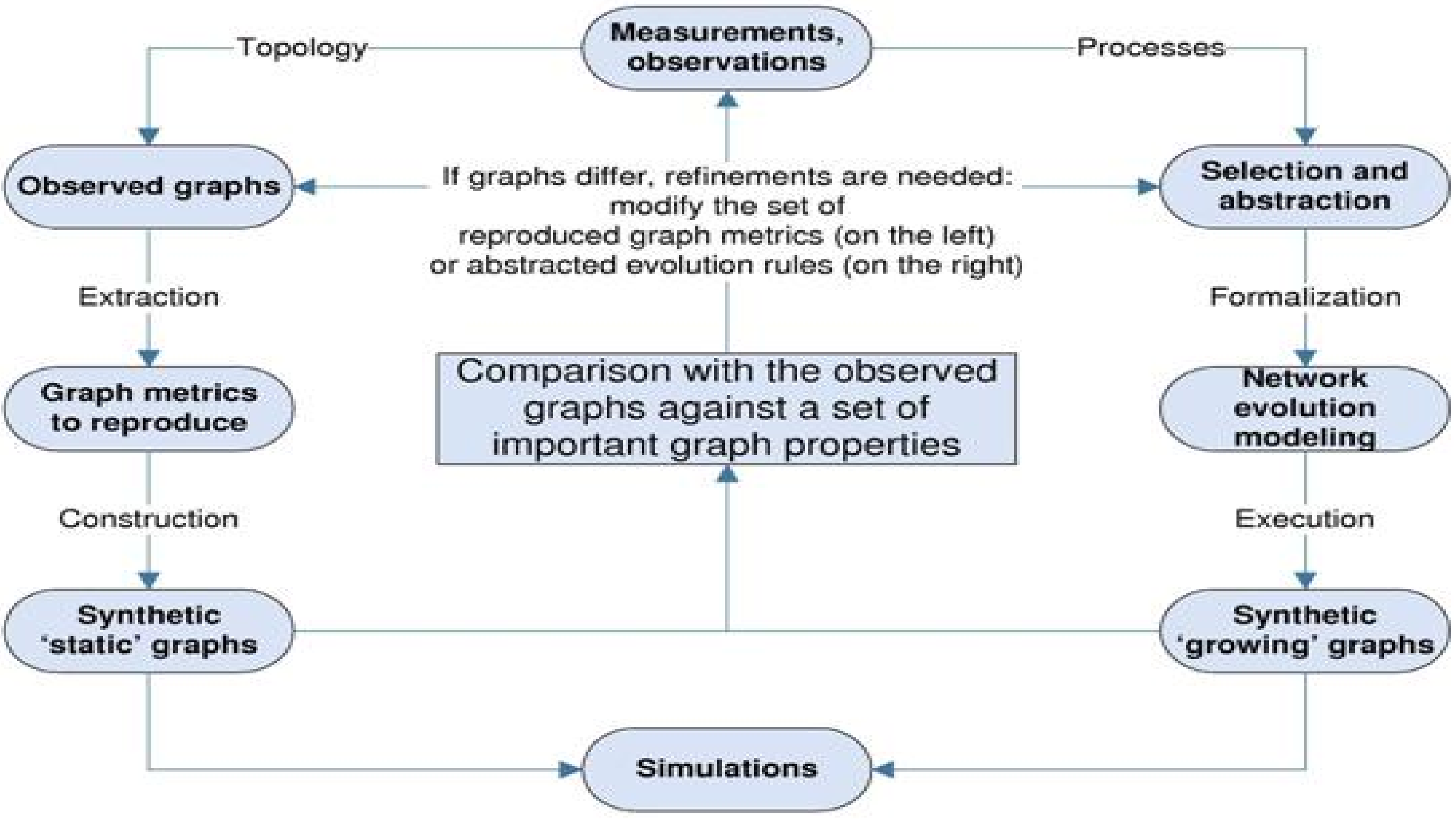
Motivation: topology analysis and generation

- New *routing* and other protocol design, development, testing, etc.
  - Analysis: performance of a routing algorithm strongly depends on topology, the recent progress in routing theory has become topology analysis
  - Generation: empirical estimation of scalability: new routing might offer X-time smaller routing tables for today but scale Y-time worse, with $Y >> X$
- Network robustness, resilience under attack, worm spreading, etc.
- Traffic engineering, capacity planning, network management, etc.
- In general: "what if", predictive power, evolution

Network topology research



Important topology metrics

- Spectrum
- Distance distribution
- Betweenness distribution
- Degree distribution
- Assortativity
- Clustering

Problems

- No way to reproduce most of the important metrics
- No guarantee there will not be any other/new metric found important

Our approach

- Look at inter-dependencies among topology characteristics
- See if by reproducing most basic, simple, but not necessarily practically relevant characteristics, we can also reproduce (capture) all other characteristics, including practically important
- Try to find the one(s) defining *all others*

Outline

- Introduction
- *dK*-*:
  - *dK*-distributions
  - *dK*-series
  - *dK*-graphs
  - *dK*-randomness
  - *dK*-explorations
- Construction
- Evaluation
- Conclusion

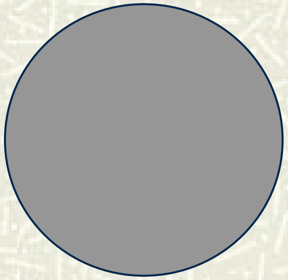
The main observation ☺

Graphs are structures of *connections* between nodes

dK-distributions as a series of graphs' *connectivity* characteristics

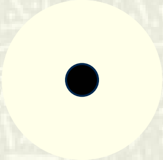
$0K$

Average degree $<k>$



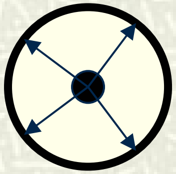
1K

Degree distribution *P(k)*



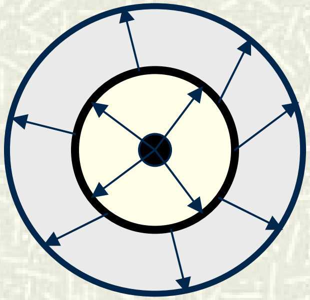
2K

Joint degree distribution $P(k_1,k_2)$

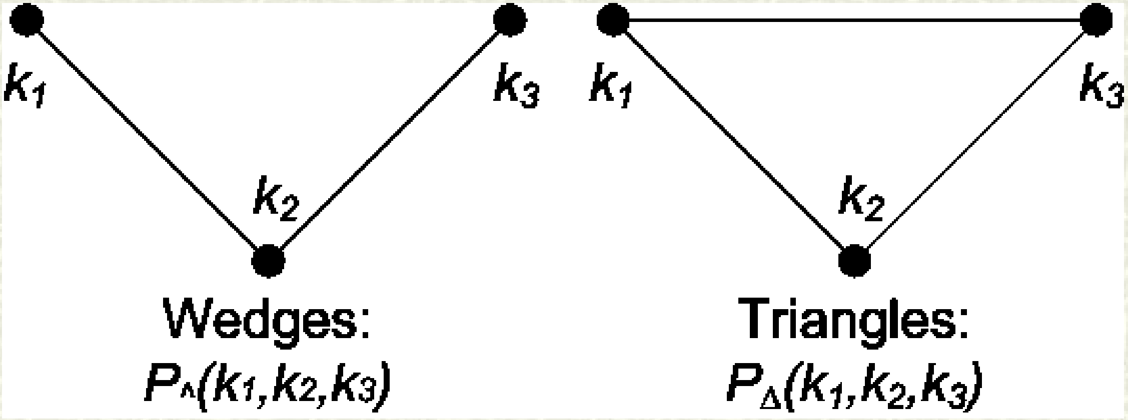


3K

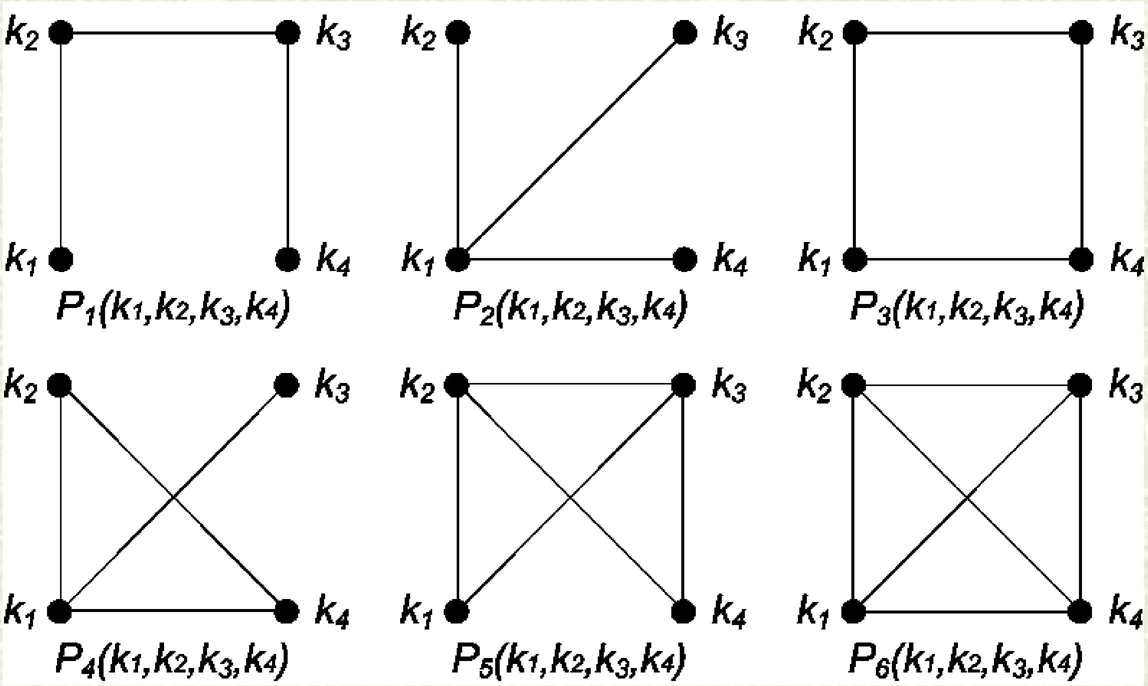
“Joint edge degree” distribution $P(k_1,k_2,k_3)$



3K, more exactly



4K



k2
k3
k1
k4
P1(k1,k2,k3,k4)
P2(k1,k2,k3,k4)
P3(k1,k2,k3,k4)
P4(k1,k2,k3,k4)
P5(k1,k2,k3,k4)
P6(k1,k2,k3,k4)

Definition of *dK*-distributions

dK-distributions are degree correlations within simple connected graphs of size *d*

Definition of dK-series P_d

Given some graph G, graph G' is said to have *property* P_d if G''s dK-distribution is the same as G's

Definition of dK-graphs

dK-graphs are graphs having property P_d

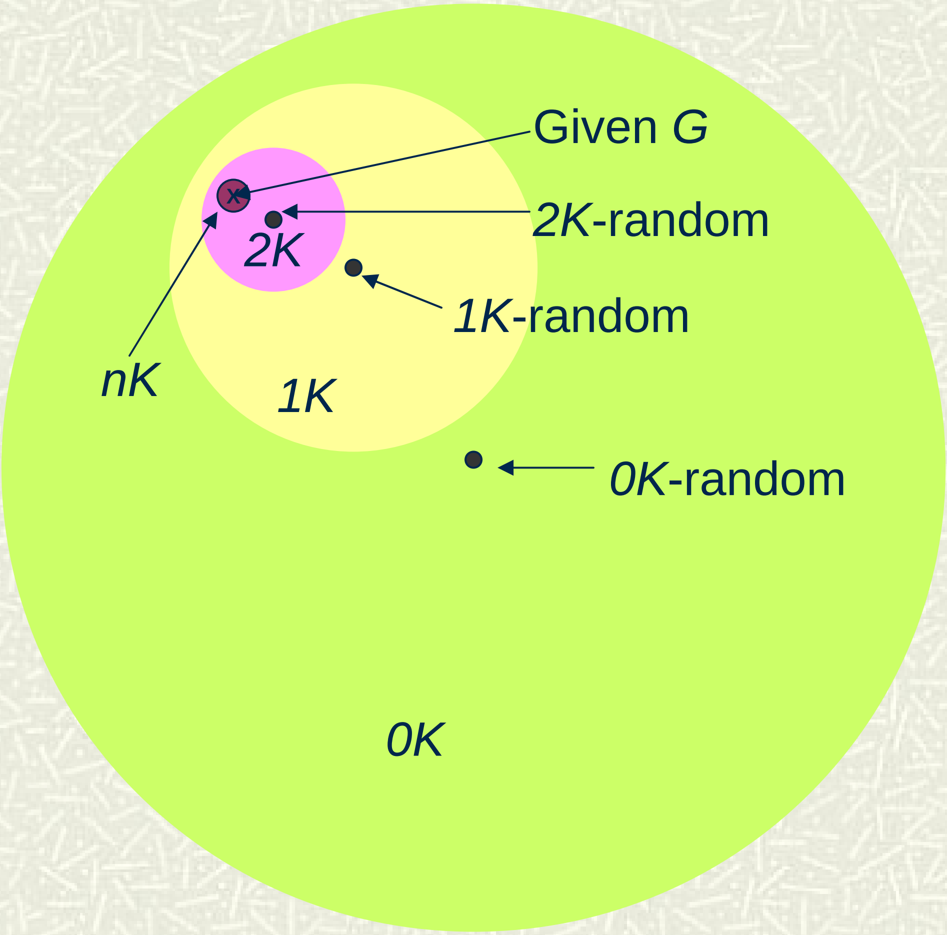
Nice properties of properties P_d

- **Constructability**: we can construct graphs having properties P_d (dK-graphs)
- **Inclusion**: if a graph has property P_d, then it also has all properties P_i, with $i < d$ (dK-graphs are also iK-graphs)
- **Convergence**: the set of graphs having property P_n consists only of one element, G itself (dK-graphs converge to G)

Convergence…

…guarantees that *all* (even not yet defined!) graph metrics can be captured by sufficiently high d

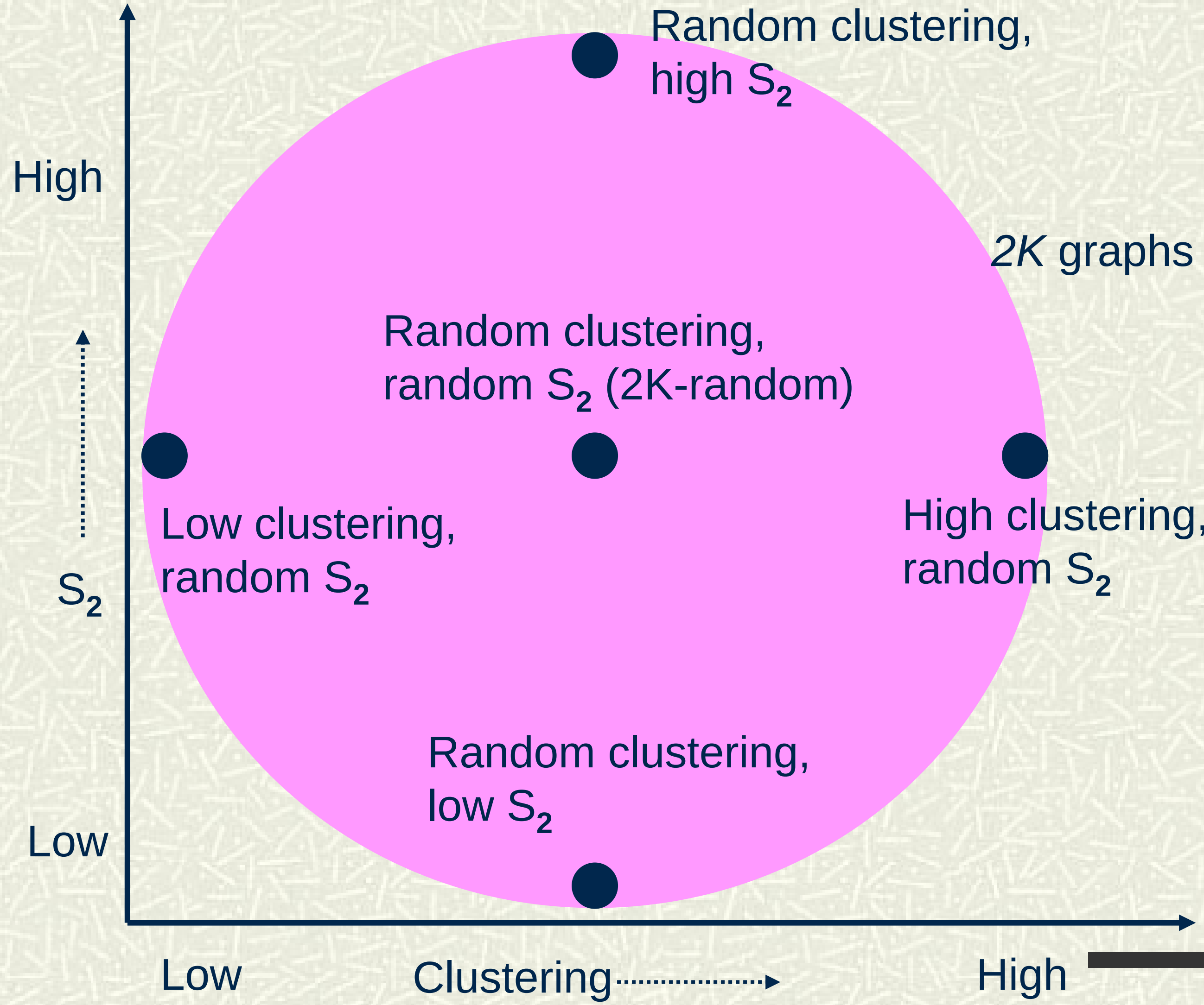
Inclusion and *dK*-randomness



dK-explorations

- To identify the minimum d, s. t. dK(-random) graphs provide a sufficiently accurate approximation of G:
- Find simple (scalar) metrics that are defined by P_{d+1} but not by P_d and construct dK-nonrandom-graphs with extreme (max or min) values of these metrics
- There are two extreme metrics of this type
  - correlations of degrees of nodes at distance d
  - concentration of d-simplices
- If differences between these dK-exotic graphs are small, then d is high enough

2*K*-exploration example



dK-summary

Tag dK	Property symbol	dK-distribution	$\mathcal{P}_d$ defines $\mathcal{P}_{d-1}$	Edge existence probability in stochastic constructions	Maximum entropy value of $(d+1)K$-distribution in dK-random graphs
$0K$	$\mathcal{P}_0$	$\bar{k}$		$p_{0K} = \bar{k}/n$	$P_{0K}(k) = e^{-\bar{k}}\bar{k}^k/k!$
$1K$	$\mathcal{P}_1$	$P(k)$	$k = \sum kP(k)$	$p_{1K}(q_1, q_2) = q_1 q_2/(n\bar{q})$	$P_{1K}(k_1, k_2) = k_1 P(k_1) k_2 P(k_2)/k^2$
$2K$	$\mathcal{P}_2$	$P(k_1, k_2)$	$P(k) = (\bar{k}/k)\sum_{k'} P(k, k')$	$p_{2K}(q_1, q_2) = (\bar{q}/n)P(q_1, q_2)/(P(q_1)P(q_2))$	See [10] for clustering in $2K$-random graphs
$3K$	$\mathcal{P}_3$	$P_\wedge(k_1, k_2, k_3)$ $P_\triangle(k_1, k_2, k_3)$	By counting edges, we get $P(k_1, k_2) \sim \sum_k \{P_\wedge(k, k_1, k_2) + P_\triangle(k, k_1, k_2)\}/(k_1 - 1) \sim \sum_k \{P_\wedge(k_1, k_2, k) + P_\triangle(k_1, k_2, k)\}/(k_2 - 1)$, where we omit normalization coefficients.		
$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$
nK	$\mathcal{P}_n$	G			

Constructability

- Introduction
- *dK*-*
- Construction
  - Stochastic
  - Pseudograph
  - Matching
  - Rewiring
    - *dK*-randomizing
    - *dK*-targeting
- Evaluation
- Conclusion

Stochastic approach

- Classical (Erdos-Renyi) random graphs are *0K*-random graph in the stochastic approach
- Easily generalizable for any *d*:
  - Reproduce the expected value of the *dK*-distributions by connecting random *d*-plets of nodes with (conditional) probabilities extracted from *G*
- Best for theory
- Worst in practice

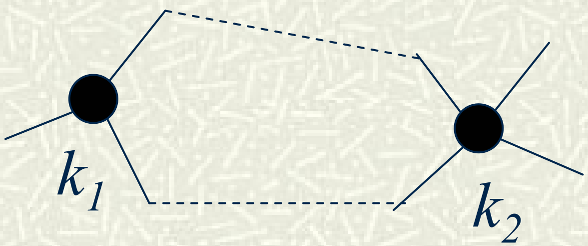
Pseudograph approach

- Reproduces dK-distributions exactly
- Constructs not necessarily connected pseudographs
- Extended for $d = 2$
- Failed to generalize for $d > 2$: d-sized subgraphs start overlap over edges at $d = 3$

Pseudograph details

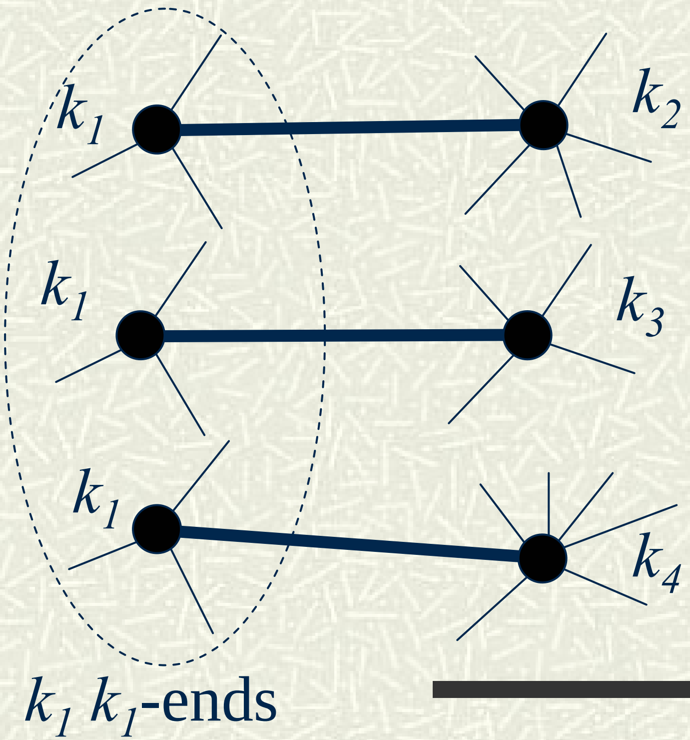
1K

1. dissolve graph into a random soup of nodes
2. crystallize it back



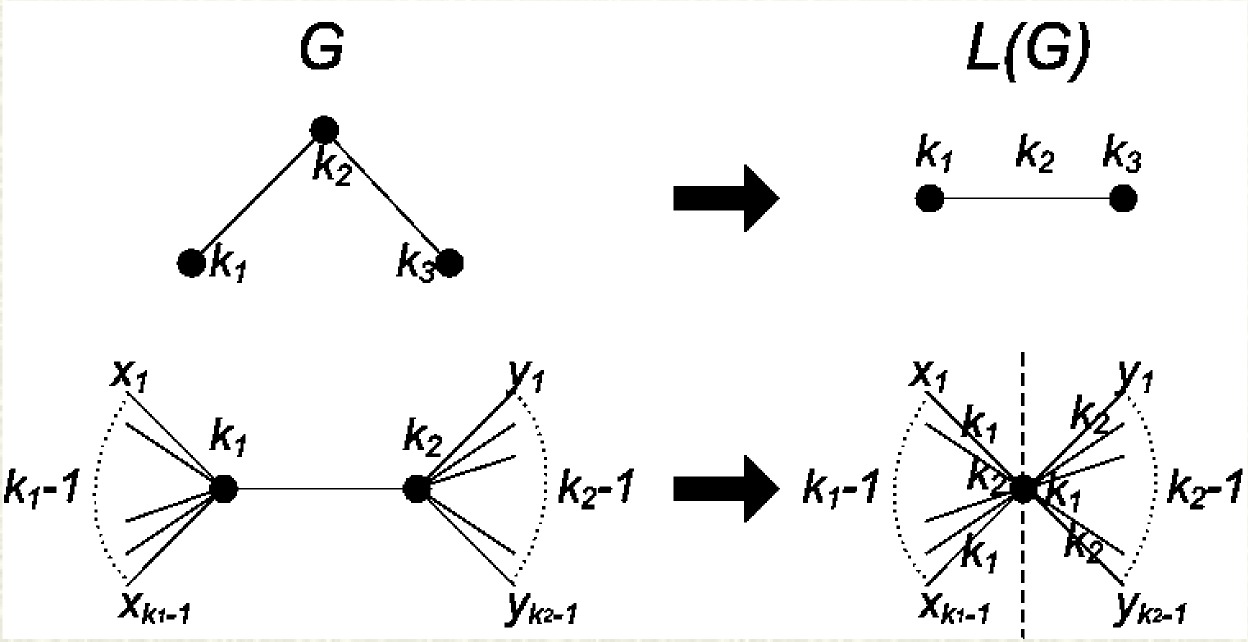
2K

1. dissolve graph into a random soup of edges
2. crystallize it back



3K-pseudograph failure

1. dissolve graph into a random soup of *d*-plets
2. cannot crystallize it back



Matching approach

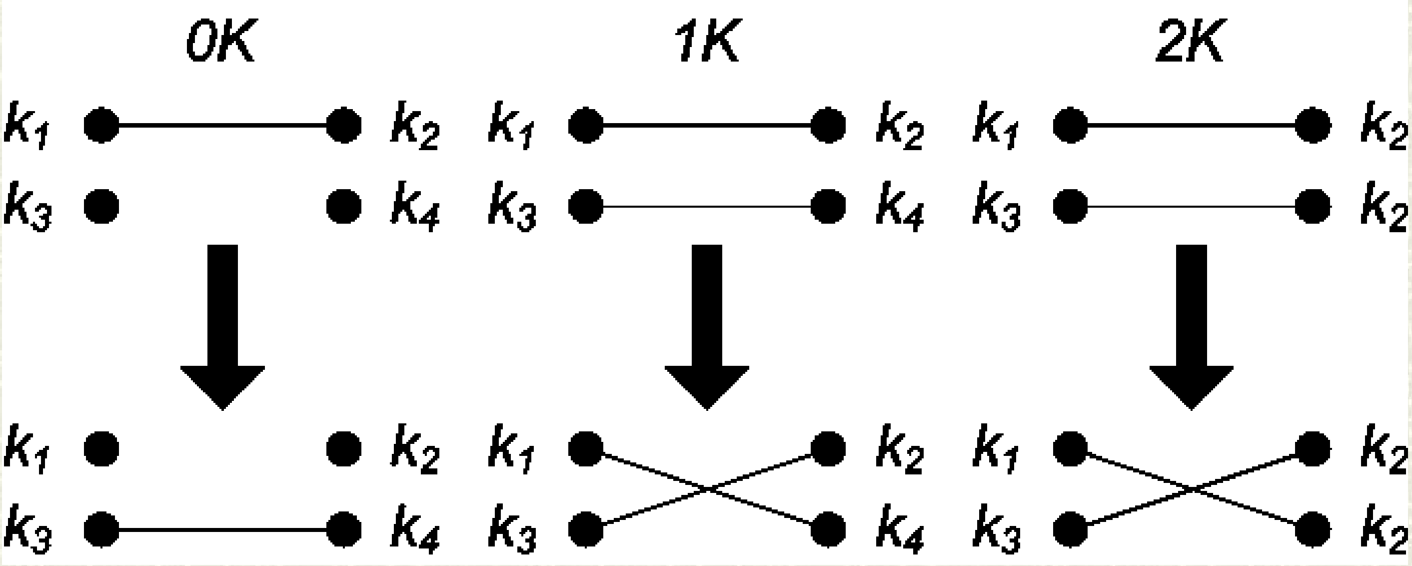
- Pseudograph + badness (loop) avoidance
- Extended for $d = 2$, but loop avoidance is difficult
- Failed to generalize for $d > 2$

Rewiring

- Generalizable for any *d*
- Works in practice

dK-randomizing rewiring

dK-preserving random rewiring



dK-targeting rewiring

- *d'K*-preserving rewiring ($d' < d$) moving graph closer to *dK*
- *dK*-distance D_d can be any non-negative scalar metric measuring the difference between the current and target values of the *dK*-distribution (e.g., the sum of squares of differences in numbers of *d*-sized subgraphs)
- Normally, accept a rewiring only if $\Delta D_d \leq 0$
- To check ergodicity:
  - accept a rewiring even if $\Delta D_d > 0$ with probability $\exp(-\Delta D_d / T)$
    - $T \to \infty$: *d'K*-randomizing rewiring
    - $T \to 0$: *dK*-targeting rewiring
  - start with a high temperature and gradually cool down the system

Outline

- Introduction
- *dK*-*
- Construction
- Evaluation
  - Algorithms
  - Topologies
    - skitter
    - HOT
- Conclusion

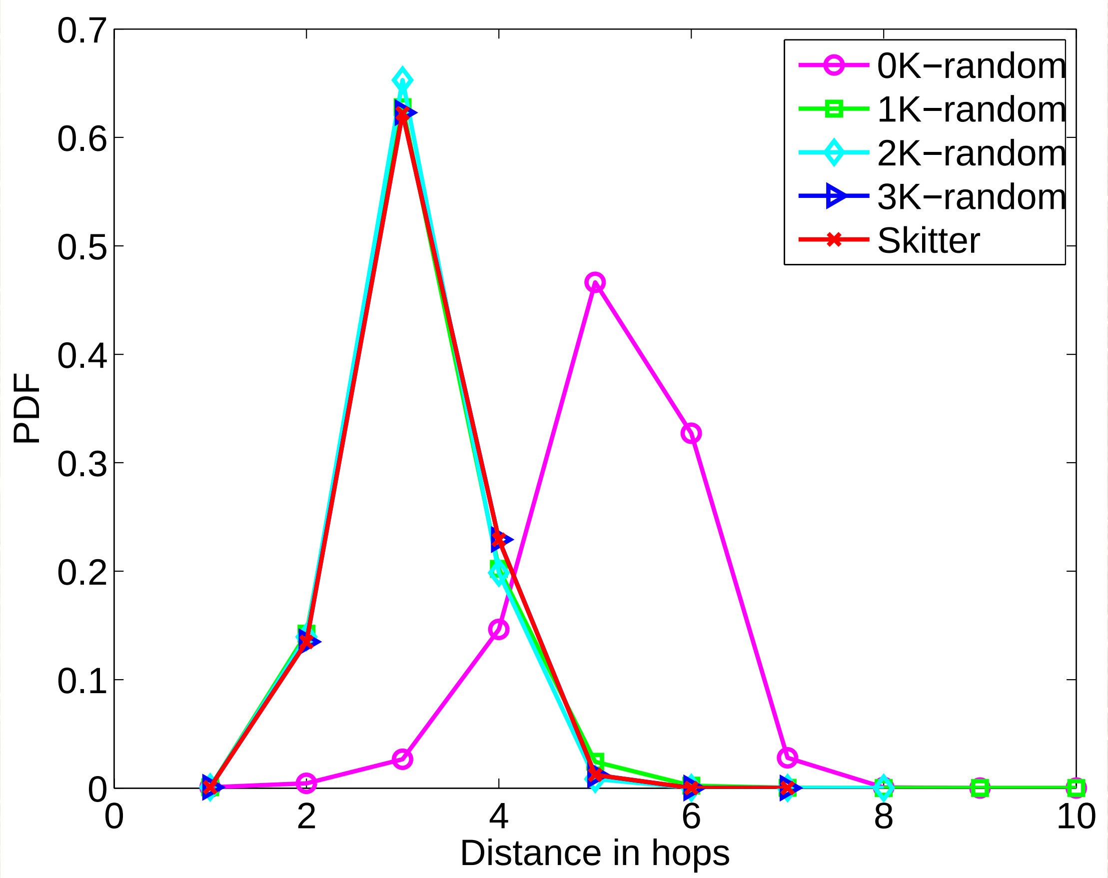
Algorithms

- All algorithms deliver consistent results for $d = 0$
- All algorithms, except stochastic(!), deliver consistent results for $d = 1$ and $d = 2$
- Both rewiring algorithms deliver consistent results for $d = 3$

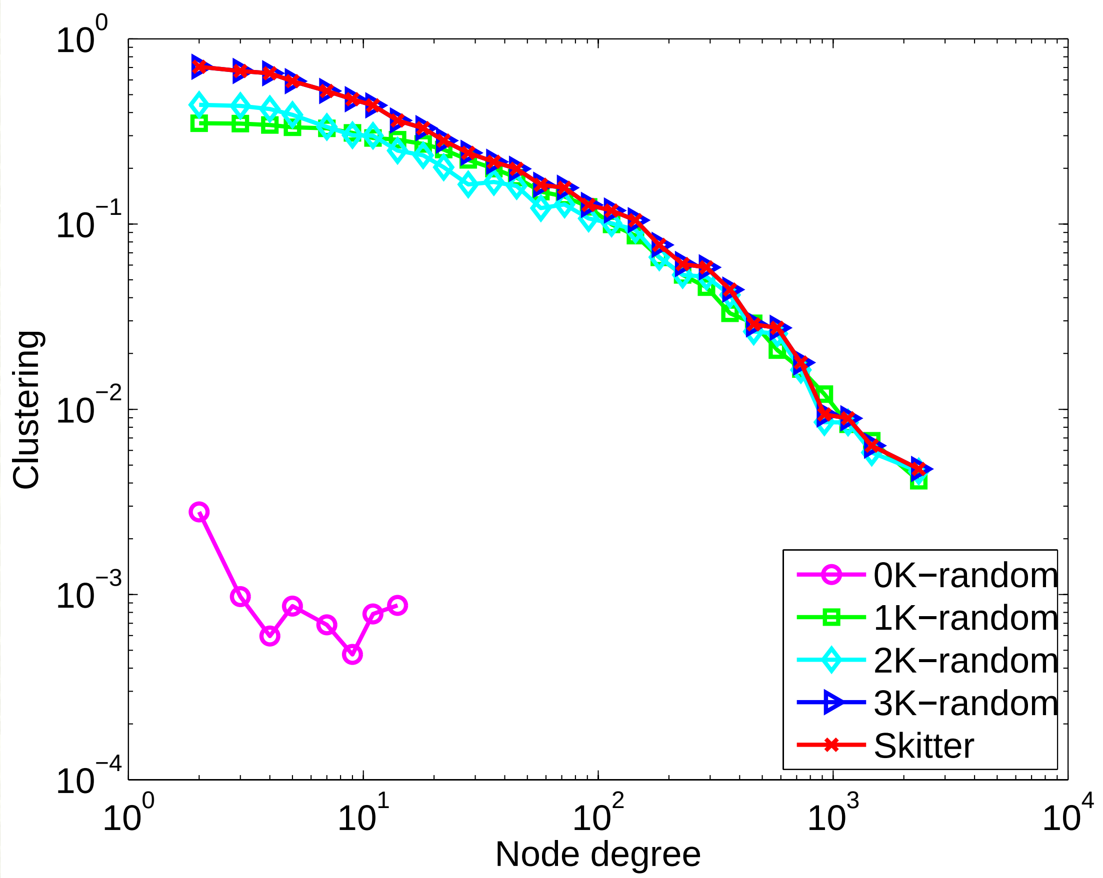
skitter scalar metrics

Metric	*0K*	*1K*	*2K*	*3K*	skitter
$<k>$	6.31	6.34	6.29	6.29	6.29
r	0	-0.24	-0.24	-0.24	-0.24
$<C>$	0.001	0.25	0.29	0.46	0.46
d	5.17	3.11	3.08	3.09	3.12
σ_d	0.27	0.4	0.35	0.35	0.37
λ_1	0.2	0.03	0.15	0.1	0.1
λ_{n-1}	1.8	1.97	1.85	1.9	1.9

skitter distance distribution



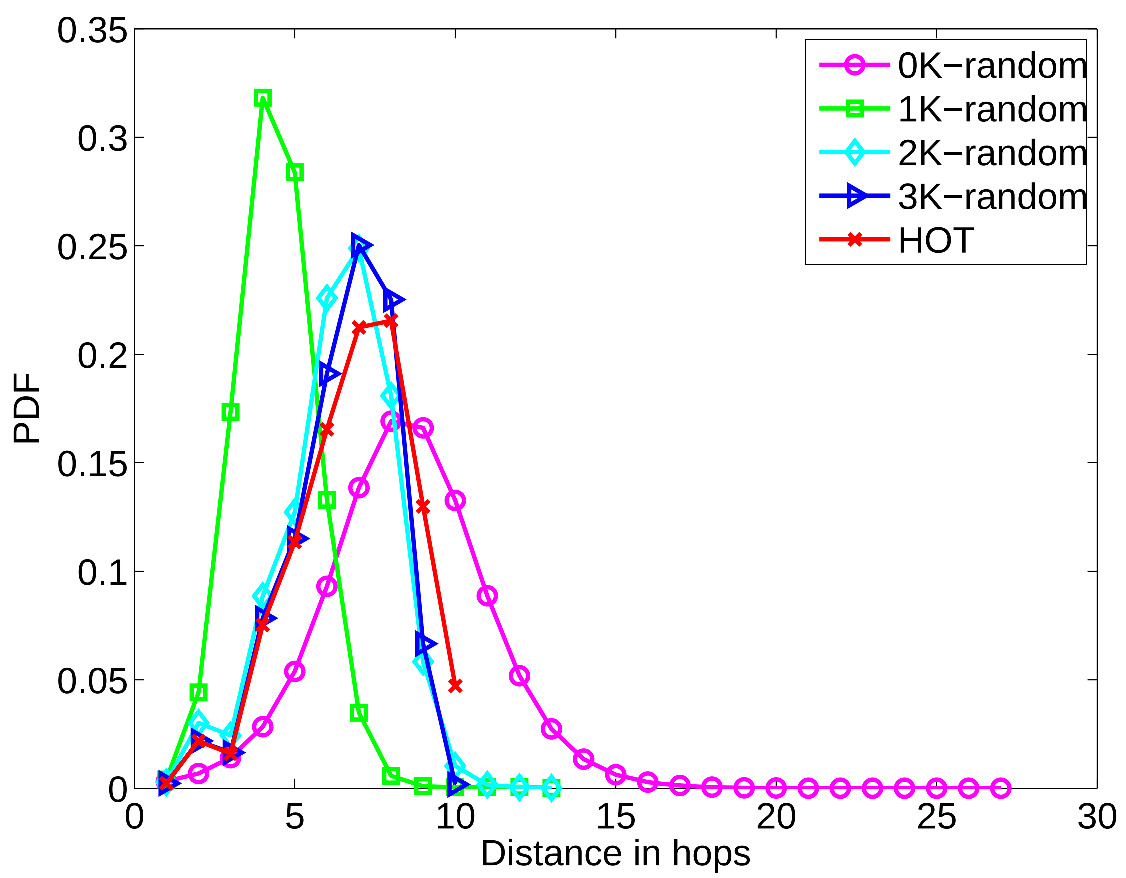
skitter clustering



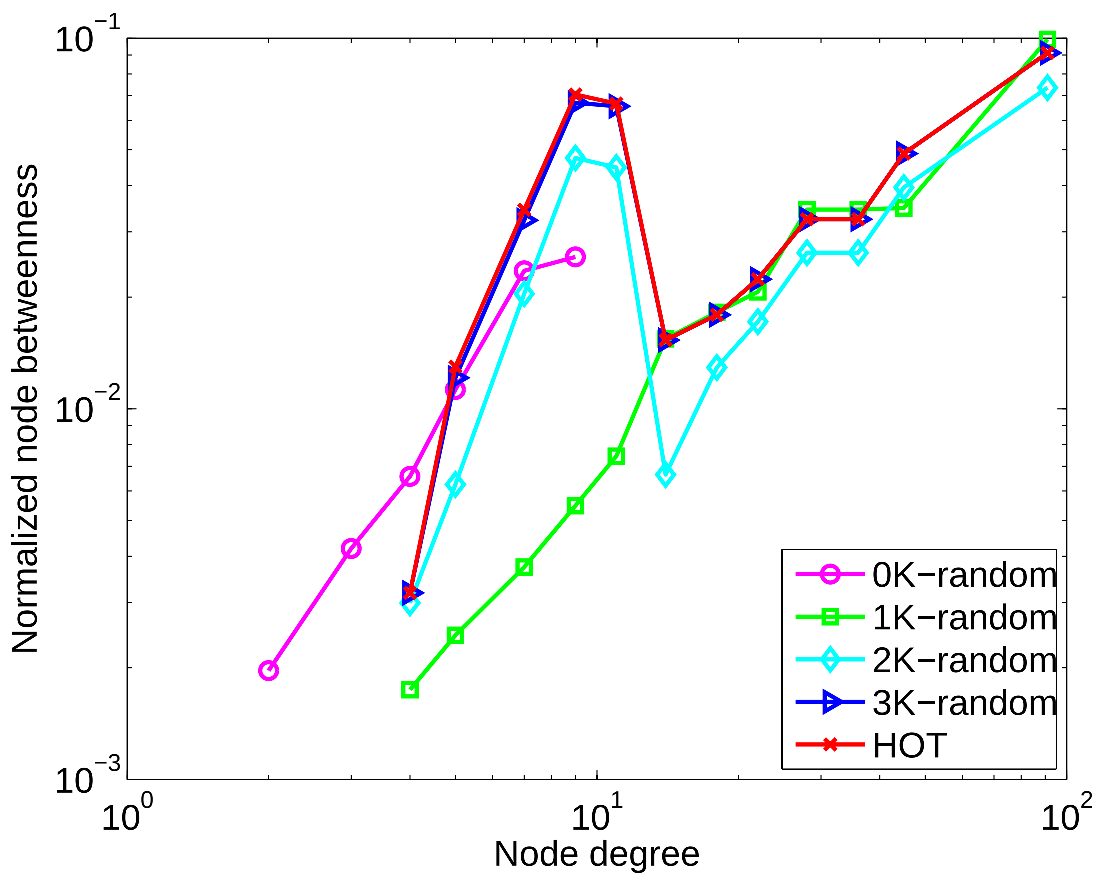
HOT scalar metrics

Metric	*0K*	*1K*	*2K*	*3K*	*HOT*
$<k>$	2.47	2.59	2.18	2.10	2.10
r	-0.05	-0.14	-0.23	-0.22	-0.22
$<C>$	0.002	0.009	0.001	0	0
d	8.48	4.41	6.32	6.55	6.81
σ_d	1.23	0.72	0.71	0.84	0.57
λ_1	0.01	0.034	0.005	0.004	0.004
λ_{n-1}	1.989	1.967	1.996	1.997	1.997

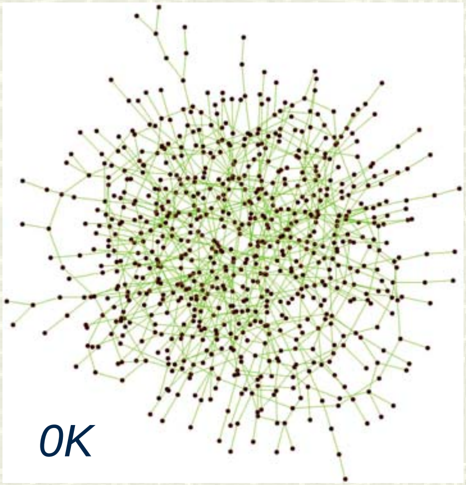
HOT distance distribution



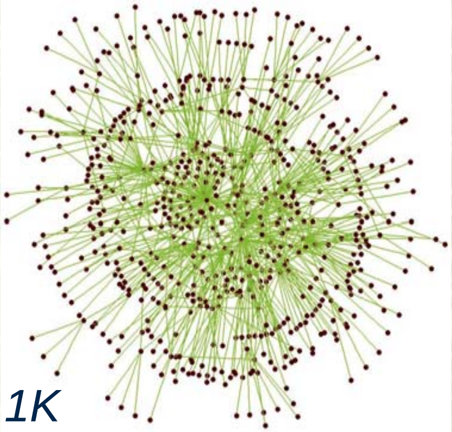
HOT betweenness distribution



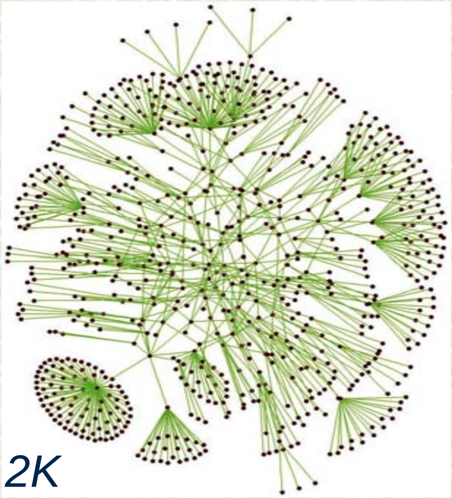
HOT *dK*-porn



0K

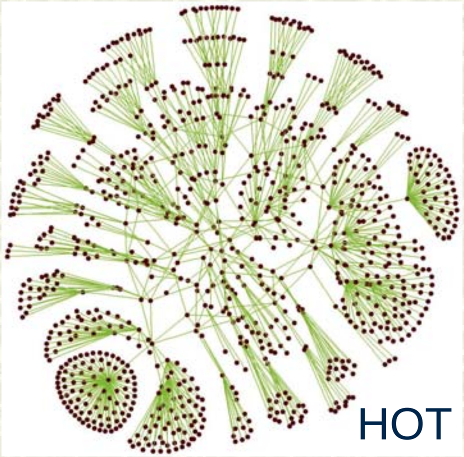


1K



2K

3K



HOT

Outline

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- *dK*-*
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Conclusions

- **Analysis**: inter-dependencies among topology metrics and connections between
  - local and global structure
  - continuous and discrete worlds
  - equilibrium and non-equilibrium models (if a topology is *dK*-random, its evolution models need to explain just the *dK*-distribution)
- **Generation**: topology generator with arbitrary level of accuracy