Application of Hyperbolic Embedding in Overlay Network Construction

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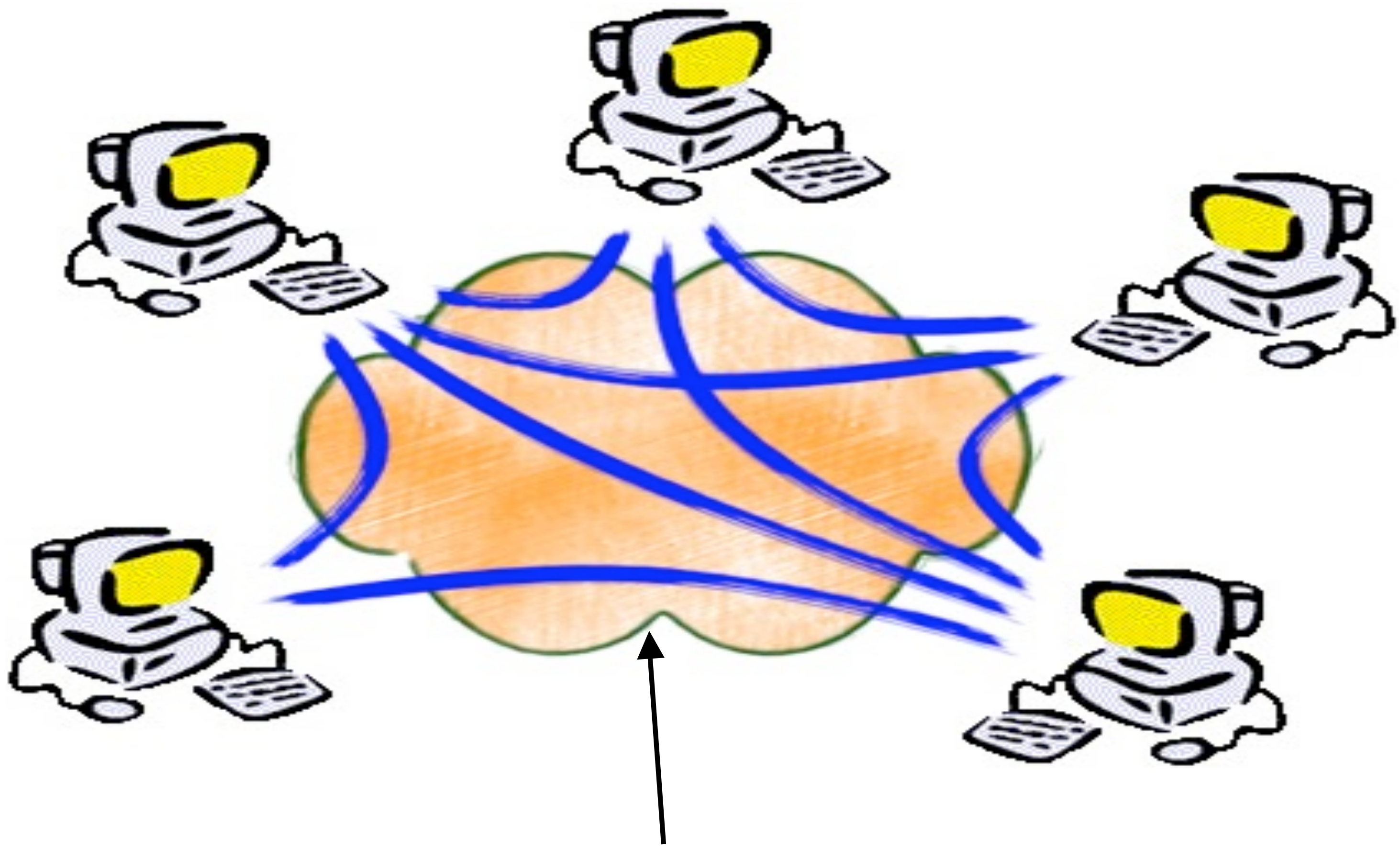
Overview

- Motivation
- A Network Model that Grows in a Hyperbolic Space
- Application to Peer-to-Peer Overlays
- Conclusion

What is Peer-to-Peer (P2P) ?

- A model of communication where every node in the network acts alike
- As opposed to the Client-Server model, where one node provides services and other nodes use the services

Example



Underlying network (i.e. router graph)

P2P/Overlay network is formed by the blue links

Advantages of P2P Networks

- Scalability
  - Every peer acts both as a Client and a Server => as demand increases, so does system capacity => scalable
  - Traditional Client-Server sharing: performance deteriorates as the number of clients increases
- No single point of failure
  - Data replication over multiple peers
  - Peers can find data without relying on centralized index servers

Types of P2P Networks

A. Unstructured (e.g. Gnutella, FastTrack)

- Overlay links are established arbitrarily when a new node joins => simple, no topology maintenance costs

- To find content: use controlled flooding of queries, random walk variations, etc. => not scalable and no guarantee that peer having content is found

B. Structured or DHTs (e.g. Chord, Kademlia)

- More structured pattern of overlay links => nodes need to maintain *up-to-date* information for a set of other nodes

- Queries are answered with very high probability

A Closer Look at Structured P2P Networks

- Main idea:

(i) Nodes are assigned unique identifiers (ids), e.g. via consistent hashing (e.g. SHA-1 on node IP address)

(ii) Data elements are also assigned unique identifiers using the same function, and are related to the node with the "closest" id

(iii) To find/store content: forward towards the node with the closest id

- Performance example: Consider a network of N peers

Chord requires: O(log N) routing information, O(log N) hops, $O(\log^2 N)$ messages per node arrival/departure

➢ *Main Problem: How to provide good performance in high churn rates*

This Talk

- Discusses the possibility of constructing overlay networks in Hyperbolic Spaces (are there any benefits?)
- To beat existing architectures, we need:

  - *Support for any churn rate with minimal information exchange*

  - Minimal routing information at nodes

  - Locate content with ~100% success

  - Shortest paths towards the peer responsible for content

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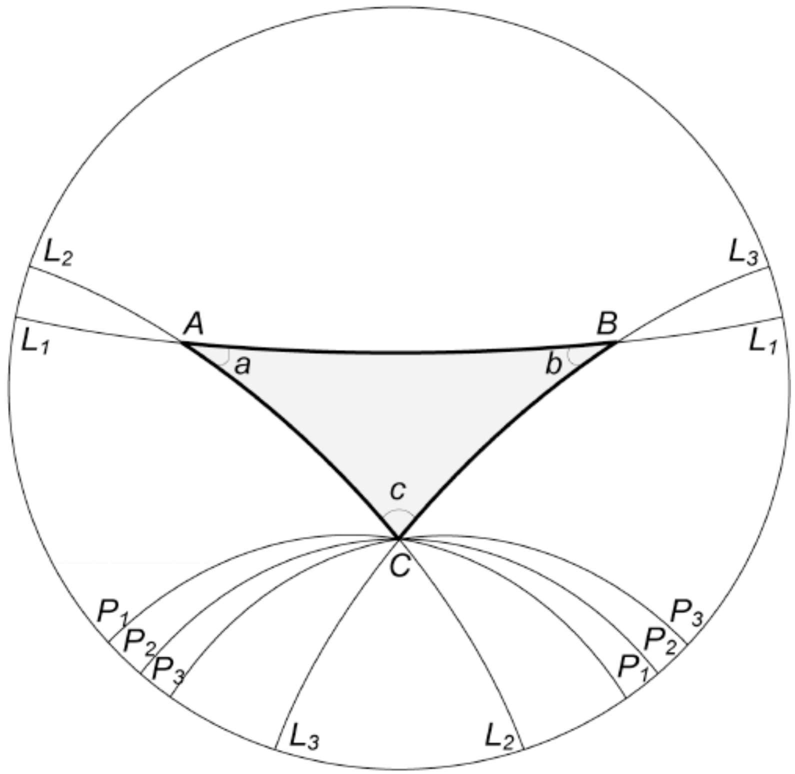
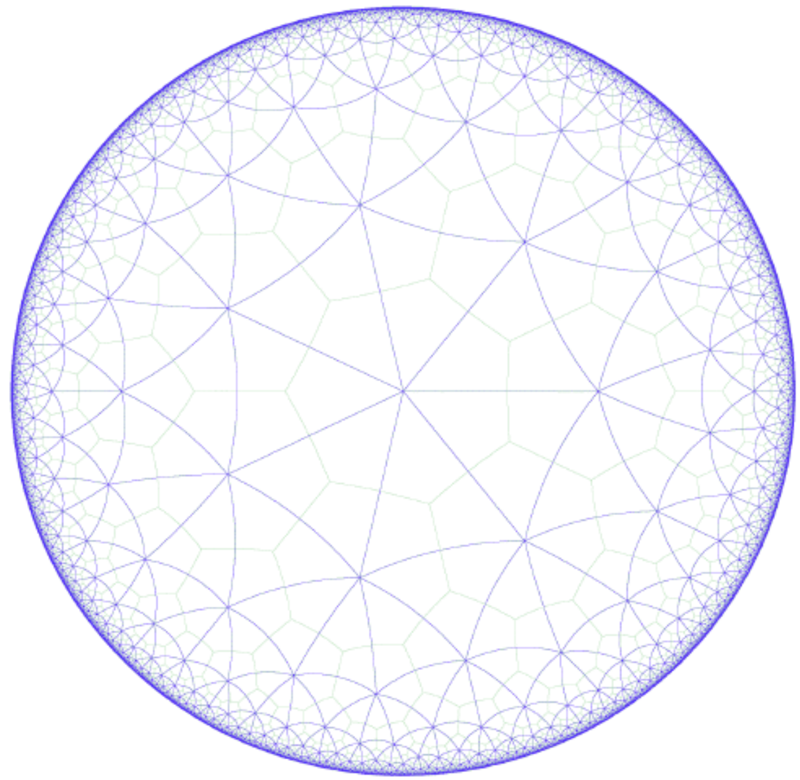
- Future Work

Hyperbolic Geometry: The Poincaré Disc Model



Poincaré disc Model



Tessellation by triangles

Constructing Scale-free Networks in the Poincaré Disc

- Perform the following operations (D. Krioukov, F. Papadopoulos, M. Boguna, A. Vahdat, 2008) :

  - Fix the disc radius to R according to $N=\kappa e^{R/2}$ where N is the number of nodes and κ used to tune the average degree to a target value

  - Assign to each node an angular coordinate θ uniformly distributed in [0, 2π]

  - Assign to each node a radial coordinate r in [0, R], with probability $\rho(r)=\alpha e^{\alpha r}/(e^{\alpha R}-1)$

  - Connect every pair of nodes whenever the *hyperbolic* distance between them is smaller than R

➢ ***Resulting graph is scale-free (power-law degree distribution and strong clustering) with exponent $\gamma = 2\alpha+1$ ($1/2 \leq \alpha \leq 1$)***

Navigation in the Poincaré Disc

- Has been shown (D. Krioukov, F. Papadopoulos, M. Boguna, A. Vahdat, 2008) :

  - Greedy routing (i.e. forward packet to the neighbor closer to destination in the hyperbolic space) has >99.9% success probability and stretch ≈1 if γ is small.

  - Above still hold in dynamic conditions (random node/link removals), ***without the need for updates***

- Question: Can we construct overlay networks in the Poincaré disc?

  - Cannot use the above model as is (assumes network size does not grow)

  - We need a ***growing model***

A Growing Model

- Initially there are 0 nodes in the network
- A new arriving node i needs to know:

  a. The current number of nodes in the network N(i)
  b. A pre-specified (constant) node density value δ, which dictates how the average node degree evolves
  c. The parameter of the node density distribution α, which determines the exponent of the degree distribution

A Growing Model (Cont.)

- To connect to the network, node i performs the following operations:

a. Selects an angular coordinate θ uniformly distributed in $[0, 2\pi]$
b. Computes the current disc radius $R(i)=(1/\alpha)\text{acosh}(1+\alpha N(i)/2\pi\delta)$
c. Selects a radial coordinate r in $[0, R(i)]$, with probability $\rho(r)=\alpha e^{\alpha r}/(e^{\alpha R(i)}-1)$
d. Connects to all nodes in the network for which their hyperbolic distance to it is smaller than $R(i)$

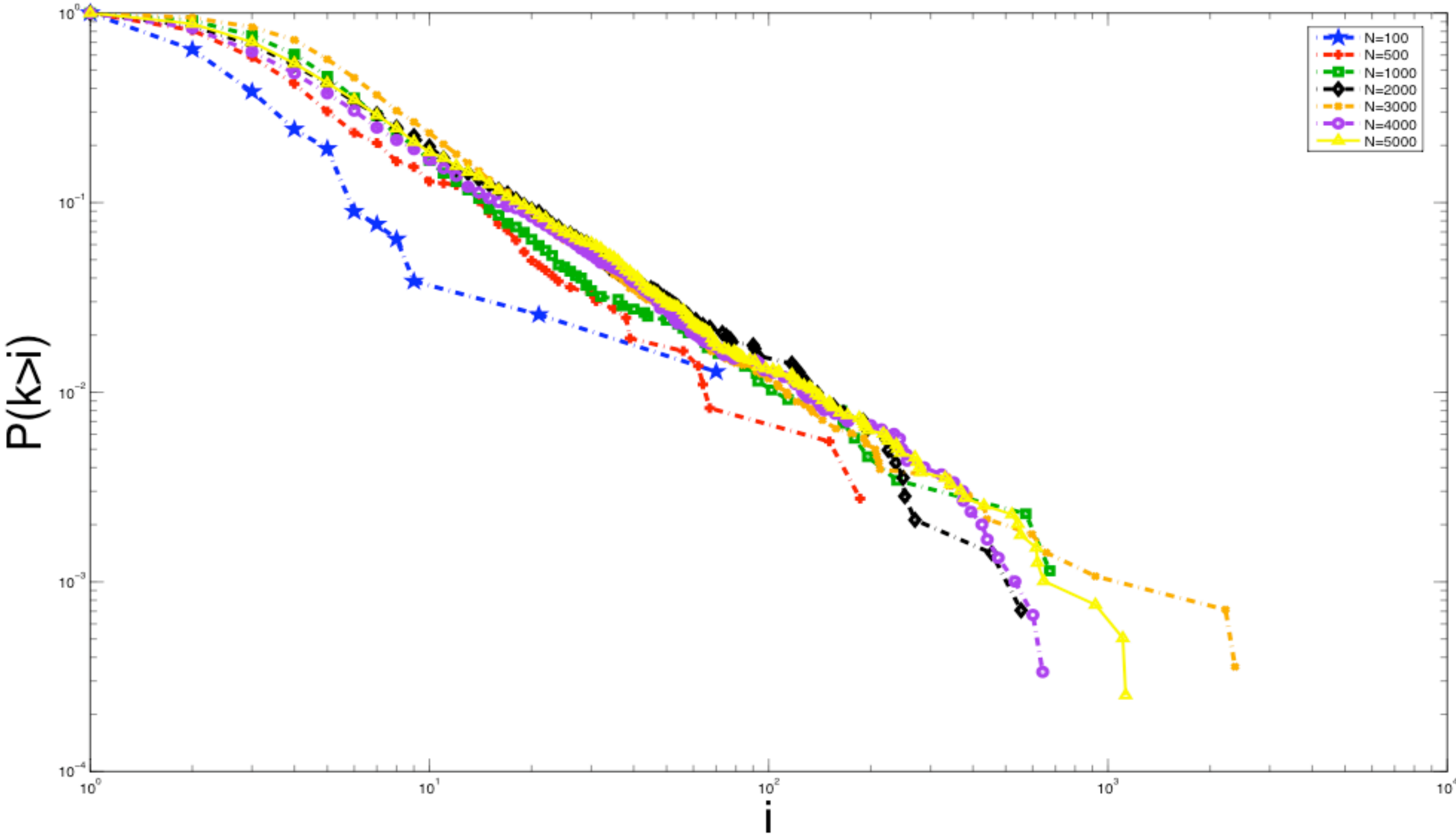
- Differences from the static model

(i) The disc grows $R(i) \sim \ln N(i)$

(ii) Average degree grows slowly $\sim \ln N(i)$

(iii) Maximum degree also grows $\sim N(i)^{1/(\gamma-1)}$

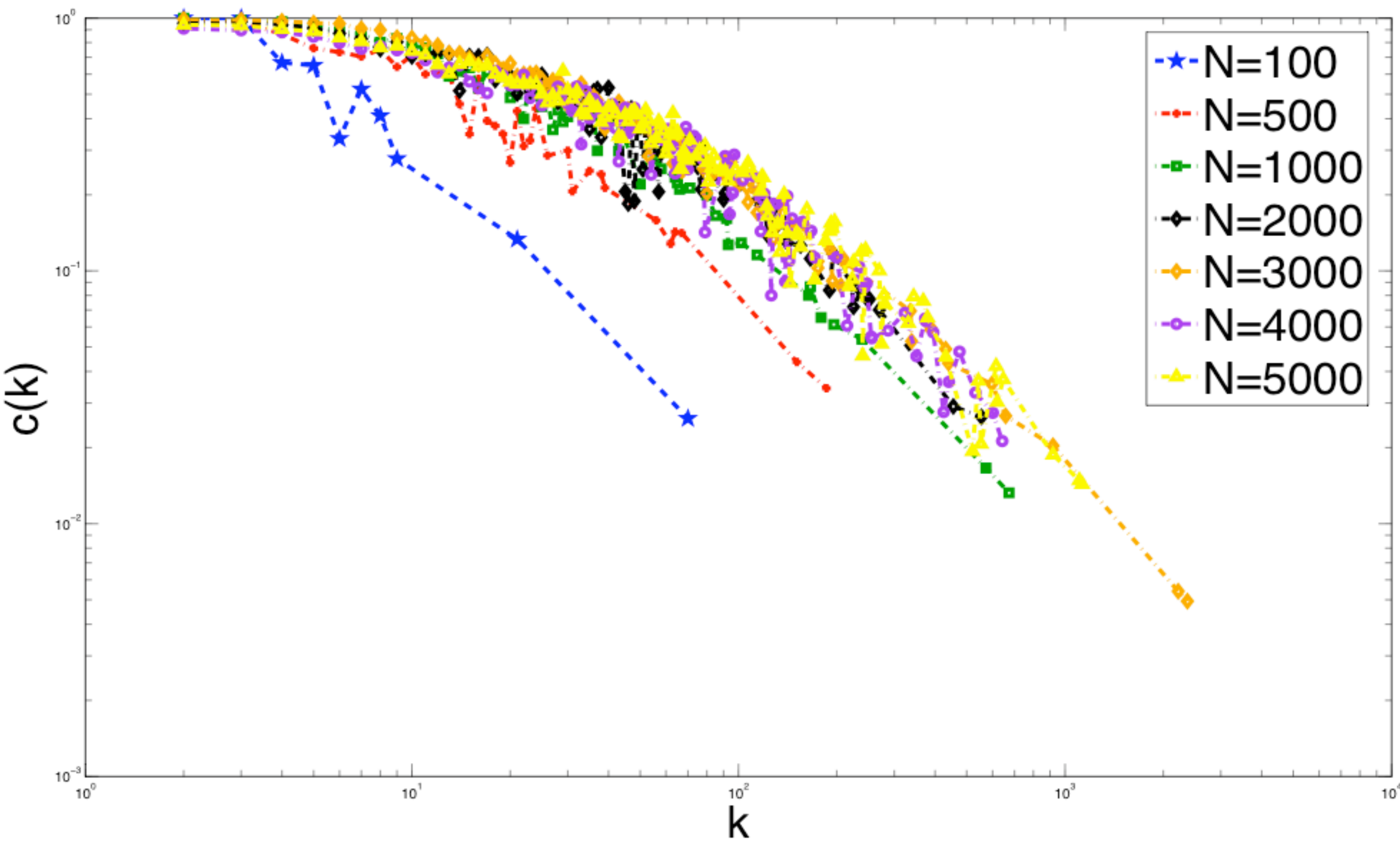
Degree Distribution



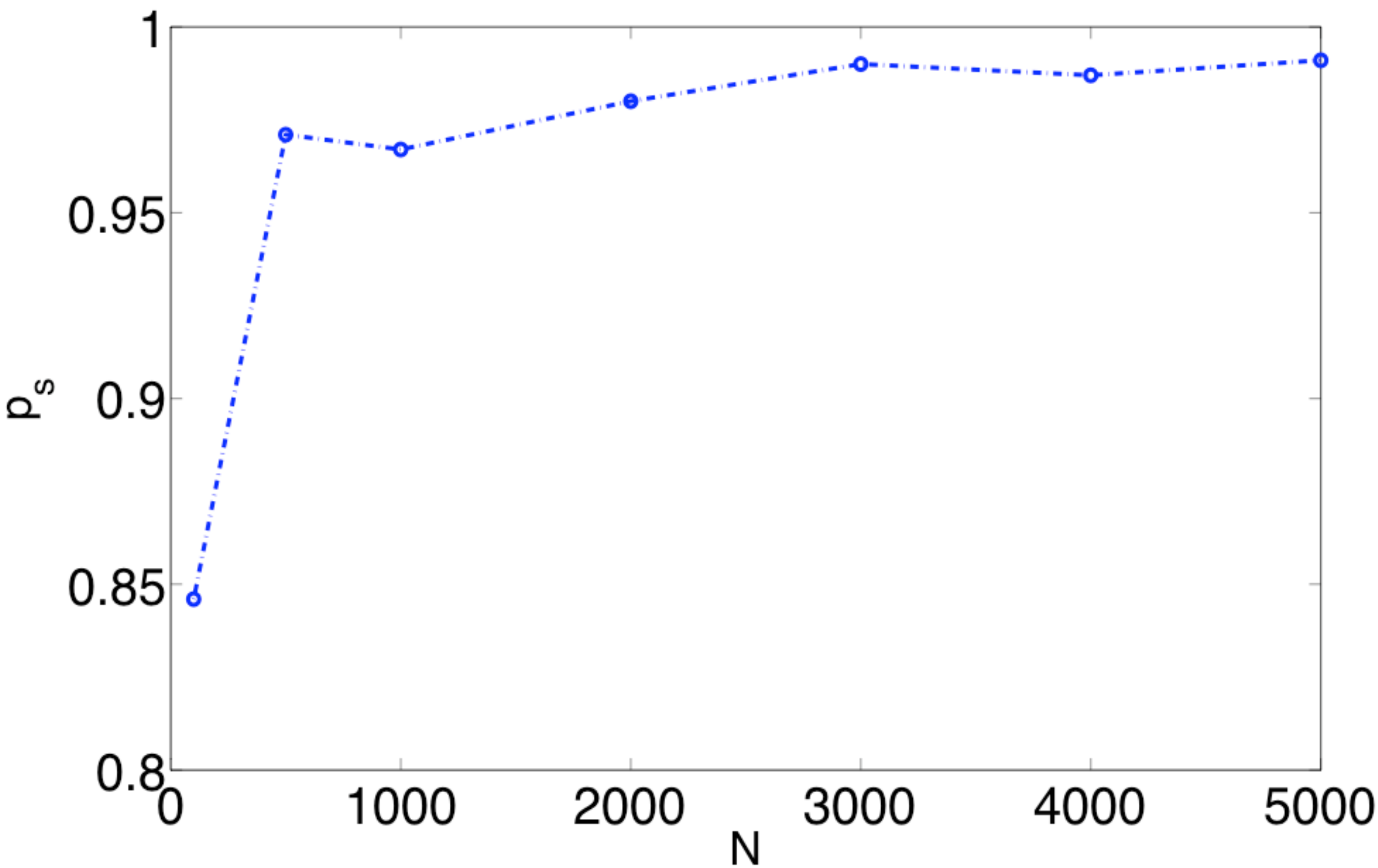
δ=0.003, α=0.6, γ≈2.2

Average degree in the range 3.9-11 as N changes from 100 to 5000

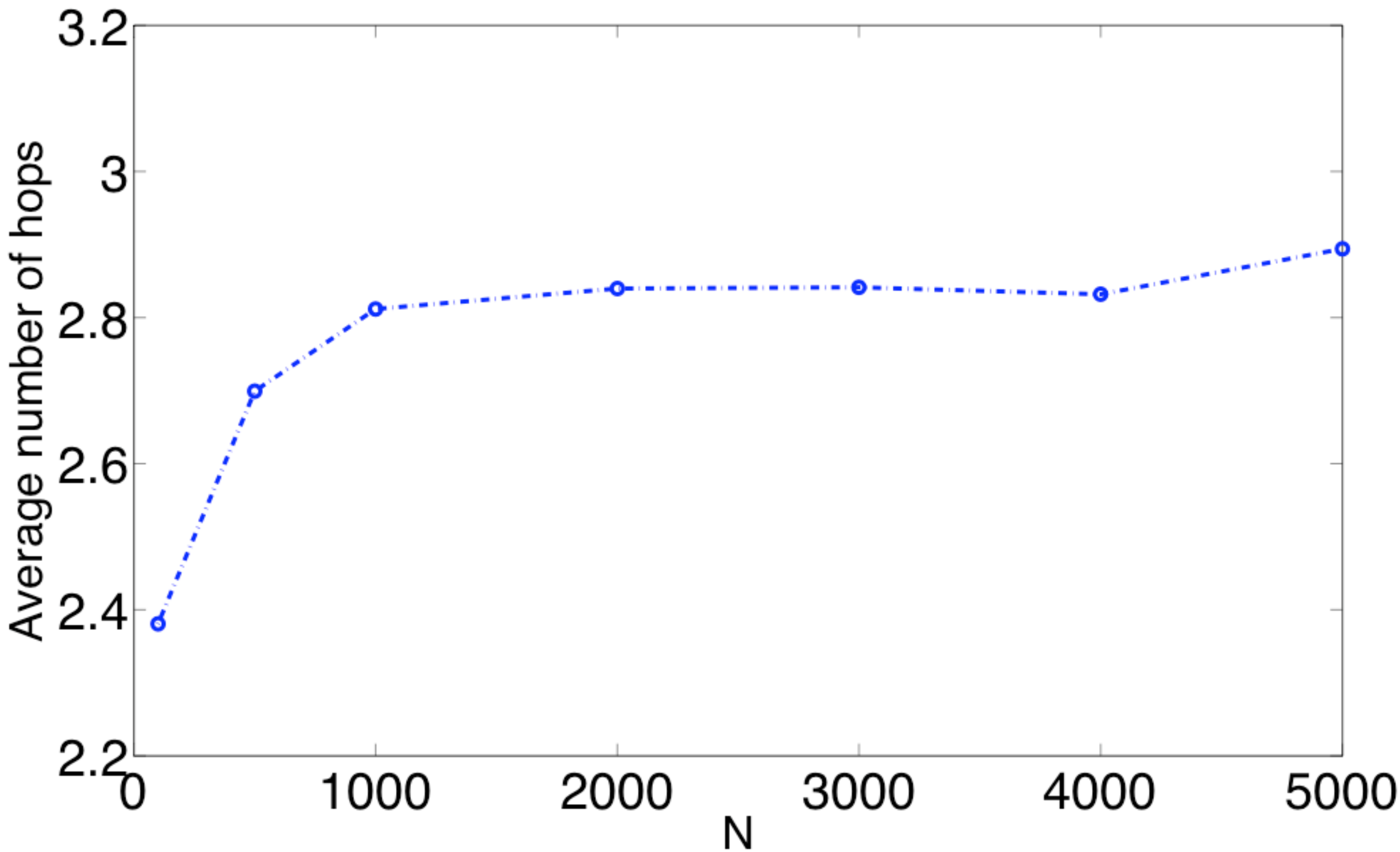
Clustering



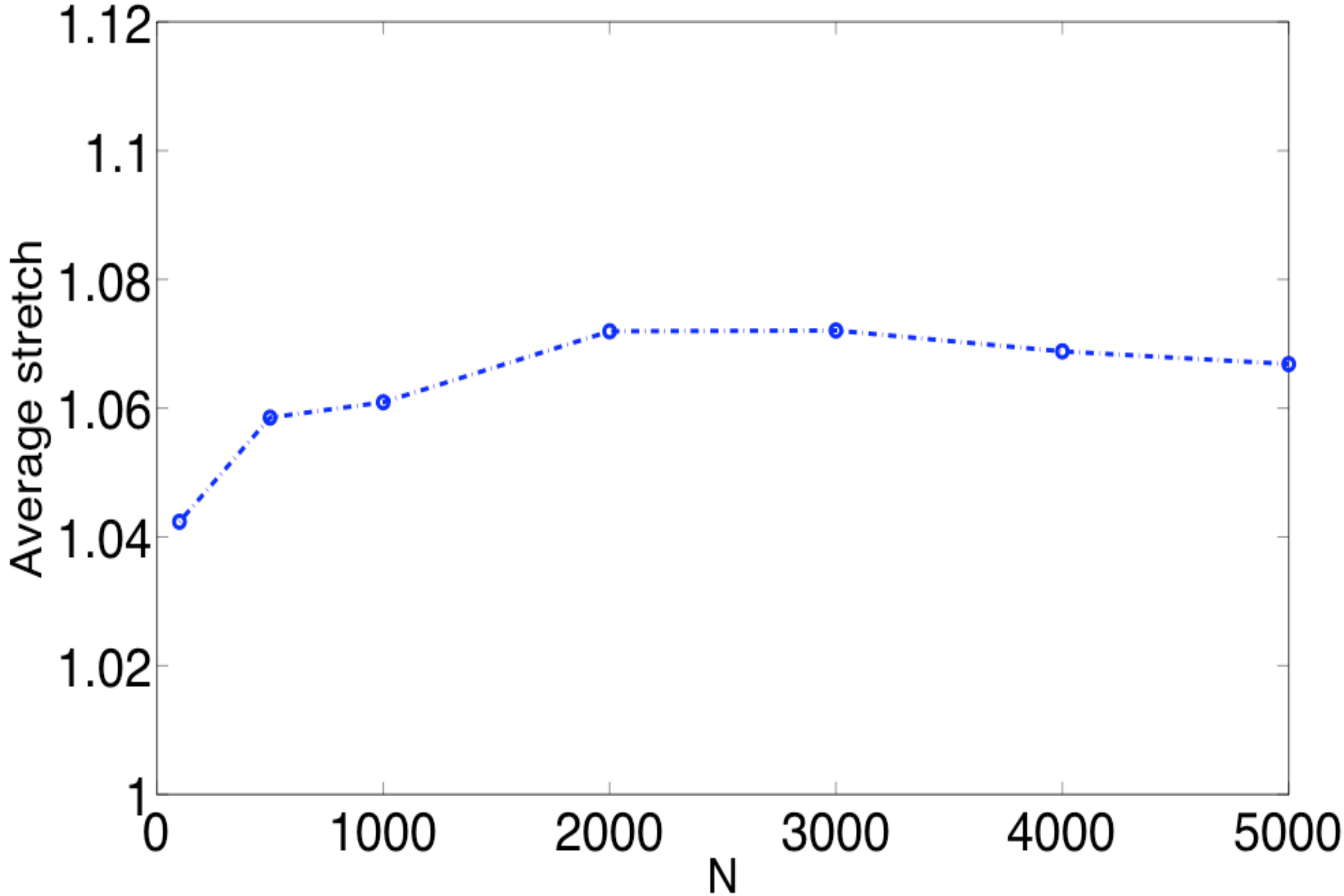
Greedy Routing Success Probability



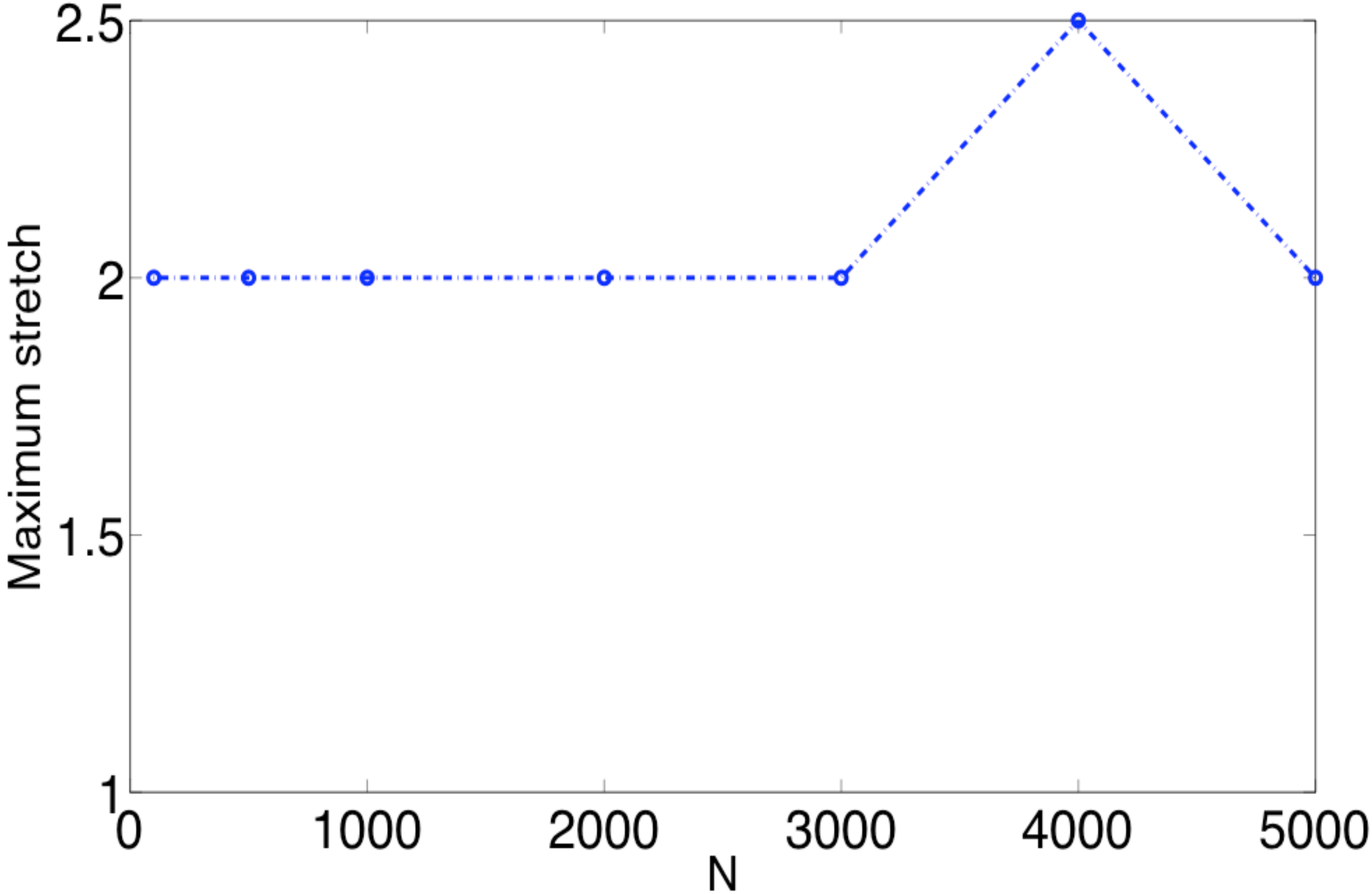
Greedy Routing Av. Number of Hops



Greedy Routing Av. Stretch



Greedy Routing Max. Stretch



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Constructing the Overlay

- **Node id**: hyperbolic coordinates (unique since continuous domain)
- Recall: Arriving node i needs to discover N(i) (to compute R(i)), and its neighbors
- Use "discovery packet"
  - Forward to highest degree neighbor
  - Neighbor writes its own and neighbors' coordinates
  - Forwards to highest degree neighbor
  - Node that sees discovery packet twice sends it back to node i
- ***Power-law graph = > the procedure terminates in very few hops***

Constructing the Overlay (Cont.)

Graph size	Discovered Graph size
69	62(13)
404	392(32)
863	831(49)
1810	1752(71)
2778	2660(86)
3744	3640(117)
4712	4586(126)

- Neighbor discovery also very efficient
- Note, R(i)~ln N(i) => very small error if estimated N(i) is not 100% accurate

Data Operations (in progress)

- **Data element's id**: also hyperbolic coordinates

- **Store operation:** at the node whose id is closest to the data id (in hyperbolic distance); How many ids/copies per data?

- Search and Retrieve: perform greedy routing with the data id as the destination

➢ *Above ensure minimal routing information and shortest path routing (search), better than existing architectures*

➢ *But, what about success ratio?*

Success ratio depends on data id(s) and number of copies

"Conjecture": We need a small number of copies to achieve 100% success ratio

Data Operations (Cont.)

- Churn rate?

  - Nodes leaving the system can assign responsibility for their data to their neighbors

  - “Better nodes” for an item arriving to the system?

Conclusion

- Have presented a network model that grows in a hyperbolic space
- Have demonstrated that the network is scale-free
- Have demonstrated that greedy routing performs exceptionally well as the network grows
- Have discussed the possibility of constructing P2P overlays using this model
- It may be possible that these overlays will outperform all existing architectures, but be aware of the “catch” (power-law degree distribution)

THANK YOU!