

# Progress in inferring business relationships between ASs

Dmitri Krioukov

[dima@caida.org](mailto:dima@caida.org)

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# Outline

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- # Overview of ongoing projects
  - # AS relationship inference
    - Motivation
    - Problems with recent heuristics
    - How we fix them
    - Future work
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# Projects

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- # Routing is dormant
  - # Comparative analysis of Internet topologies (with P. Mahadevan, UCSD)
  - # Internet topology evolution modeling (with R. Liu, UCLA)
  - # Improving AS business relationship inference heuristics (with X. Dimitropoulos, Georgia Tech, and B. Huffaker, CAIDA)
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# Motivation

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## # Practical (providers, vendors, government)

- Money/power flow
- Traffic flow
- Network robustness

## # Theoretical (research community)

- Routing  $\Leftarrow$
  - Topology  $\Leftarrow$
  - Modeling  $\Leftarrow$
  - Validation (real data)
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# Previous results: Gao and SARK

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- # L. Gao. On inferring Autonomous System relationships in the Internet. *ToN 2001*. (Gao)
  - BGP policies  $\Rightarrow$  (in)valid paths
  - AS degree-based heuristic
  - Too many invalid paths
- # L. Subramanian, et al. Characterizing the Internet hierarchy from multiple vantage points. *INFOCOM 2002*. (SARK)
  - Combinatorial optimization to minimize the number of invalid paths (ToR problem)
  - Heuristic to solve it

# Previous results: DPP and EHS

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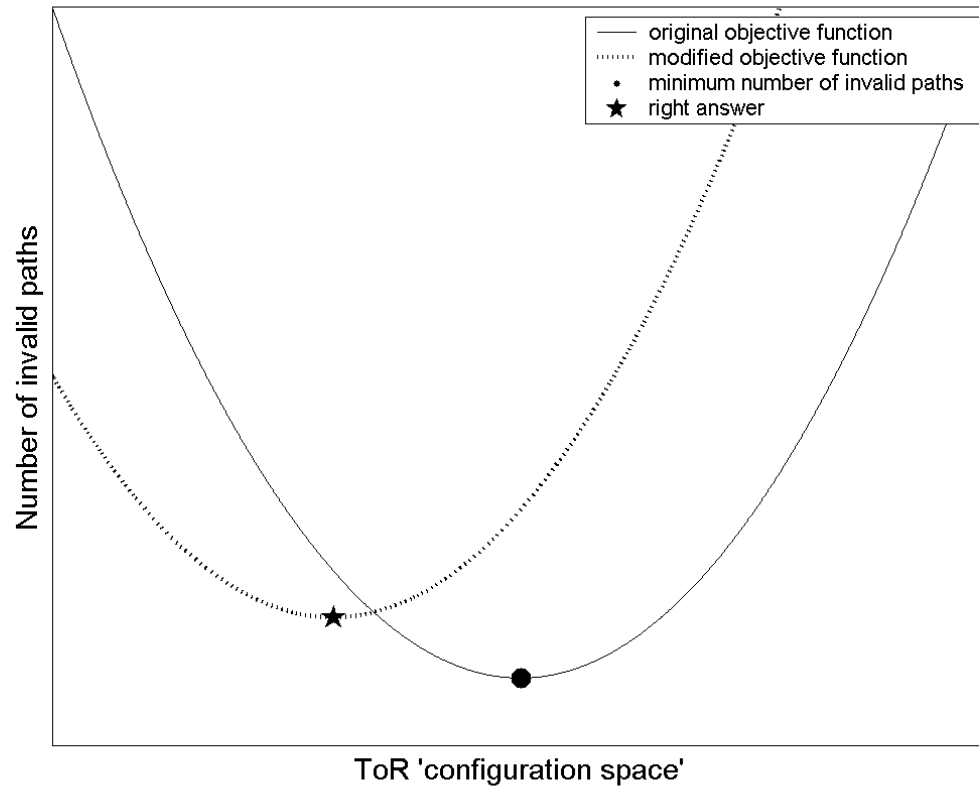
- # G. Di Battista, et al., Computing the types of the relationships between Autonomous Systems, *INFOCOM, 2003*, (DPP); **and** T. Erlebach, et al., Classifying customer-provider relationships in the Internet, *IASTED CCN, 2002*, (EHS).
  - No peering can be inferred in ToR
  - ToR is NP- and APX-complete
  - More rigorous approach to find an approximate solution
  - Smaller number of invalid paths (than in SARK)
  - Induced AS hierarchies are incorrect

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# Idea at the high level



Objective function adjustment

# ToR

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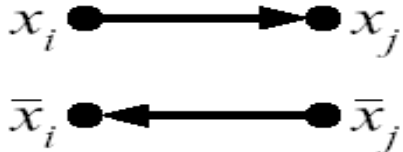


- # Given a set of BGP paths  $P$ ,
- # Extract the undirected AS-level graph  $G$ .
  - Every edge in  $G$  is a link between pair of ASs.
- # Assuming edge direction is from customer to provider,
- # Direct all edges in  $G$  ( $2^m$  combinations),
- # Inducing direction of edges in  $P$ ,
- # Such that the number of invalid paths in  $P$  is minimized.
  - Invalid path is a path containing a provider-to-customer link followed by customer-to-provider link

# ToR and MAX2SAT

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- # Split all paths in  $P$  into pairs of adjacent links (involving triplets of nodes)
- # Perform mapping...

# Mapping to MAX2SAT

Edges in $P$	2SAT clause	Edges in $G_{2SAT}$
	$x_i \vee x_j$	
	$x_i \vee \bar{x}_j$	
	$\bar{x}_i \vee x_j$	
	$\bar{x}_i \vee \bar{x}_j$	

# Two 2SAT observations

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- # All clauses can be satisfied (all paths can be made valid) if there is no variable  $x_i$  belonging with its negation to the same SCC in  $G_{2SAT}$  (conflict variable/edge)
  - SCC (strongly connected component) is a set of mutually reachable nodes in a directed graph
- # Proper direction of non-conflict edges can be done via topological sorting in  $G_{2SAT}$  (if the variable negation is before the variable itself, then the variable is *true*, and vice versa)
  - Topological sorting is a natural ordering of nodes in directed acyclic graphs

# MAX2SAT: DPP vs. EHS

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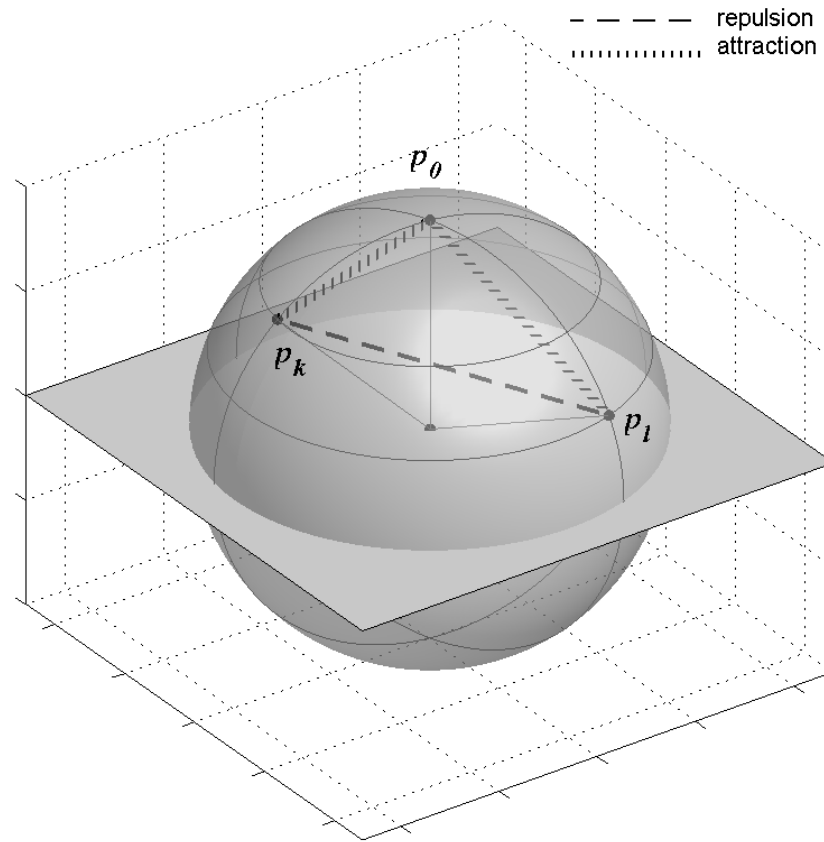
- # If  $P$  is large, not all paths (clauses) can be made valid (satisfied): 2SAT  $\Rightarrow$  MAX2SAT
- # DPP: find the maximum subset of paths that can all be made valid
- # EHS: use known algorithms to approximate MAX2SAT
  - SDP (semidefinite programming) relaxation (with certain twists) delivers approximation ratio of 0.940
  - Inapproximability ratio is 0.954

# SDP relaxation to MAX2SAT

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$$\begin{aligned} \max \quad & \frac{1}{4} \sum_{k,l=1}^{2m_1} w_{kl} (3 + v_0 \cdot v_k + v_0 \cdot v_l - v_k \cdot v_l) \\ \text{s.t.} \quad & v_0 \cdot v_0 = v_k \cdot v_k = 1, \quad v_i \cdot v_{m_1+i} = -1, \\ & k = 1 \dots 2m_1, \quad i = 1 \dots m_1. \end{aligned}$$

# Physical interpretation



# Gains and losses

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## # What's good

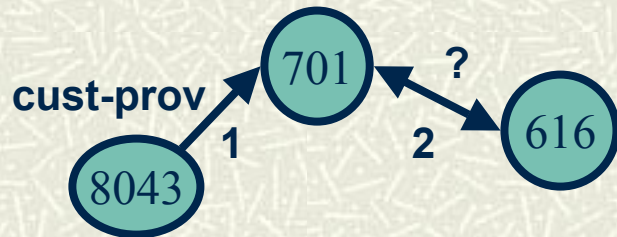
- Extremely small number of invalid paths (*99.7%*)

## # What's bad

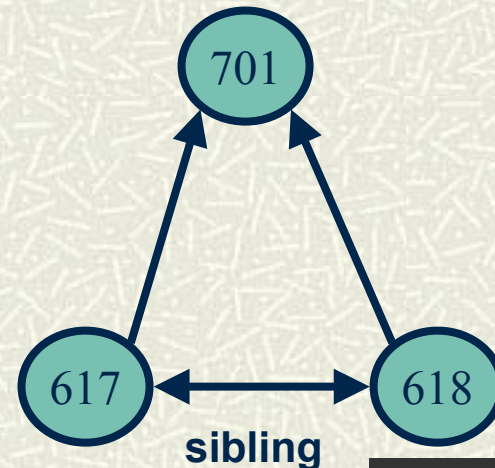
- Skewed/incorrect AS hierarchies: several small ASs are inferred as providers of large ISPs
  - But why!?
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# Causes of the problem and their resolutions

- **Case 1:** some edges can be directed any way without causing invalid paths
- **Fix:** introduce additional incentive to direct edge along the node degree gradient



- **Case 2:** trying to infer sibling links leads to proliferation of error
- **Fix:** try to discover sibling links using the WHOIS database



# Case 1:

## Multiobjective optimization

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# Maximize number of invalid paths:

- 2-link clauses  $w_{kl}(x_k \vee x_l)$

# Direct along the node degree gradient:

- 1-link clauses  $w_{kk}(x_k \vee x_k)$

# Final form of the generalized problem formulation

$$\begin{aligned} \max \quad & \frac{1}{4} \sum_{k,l=1}^{2m_1} w_{kl} (3 + v_0 \cdot v_k + v_0 \cdot v_l - v_k \cdot v_l) \\ \text{s.t.} \quad & v_0 \cdot v_0 = v_k \cdot v_k = 1, \quad v_i \cdot v_{m_1+i} = -1, \\ & k = 1 \dots 2m_1, \quad i = 1 \dots m_1. \end{aligned}$$

$$w_{kl}(\alpha) = \begin{cases} c_2 \alpha & \text{if } \{kl\} \in P, \\ c_1 (1 - \alpha) f(d_k^-, d_k^+) & \text{if } k = l \leq m_1, \\ 0 & \text{otherwise.} \end{cases}$$

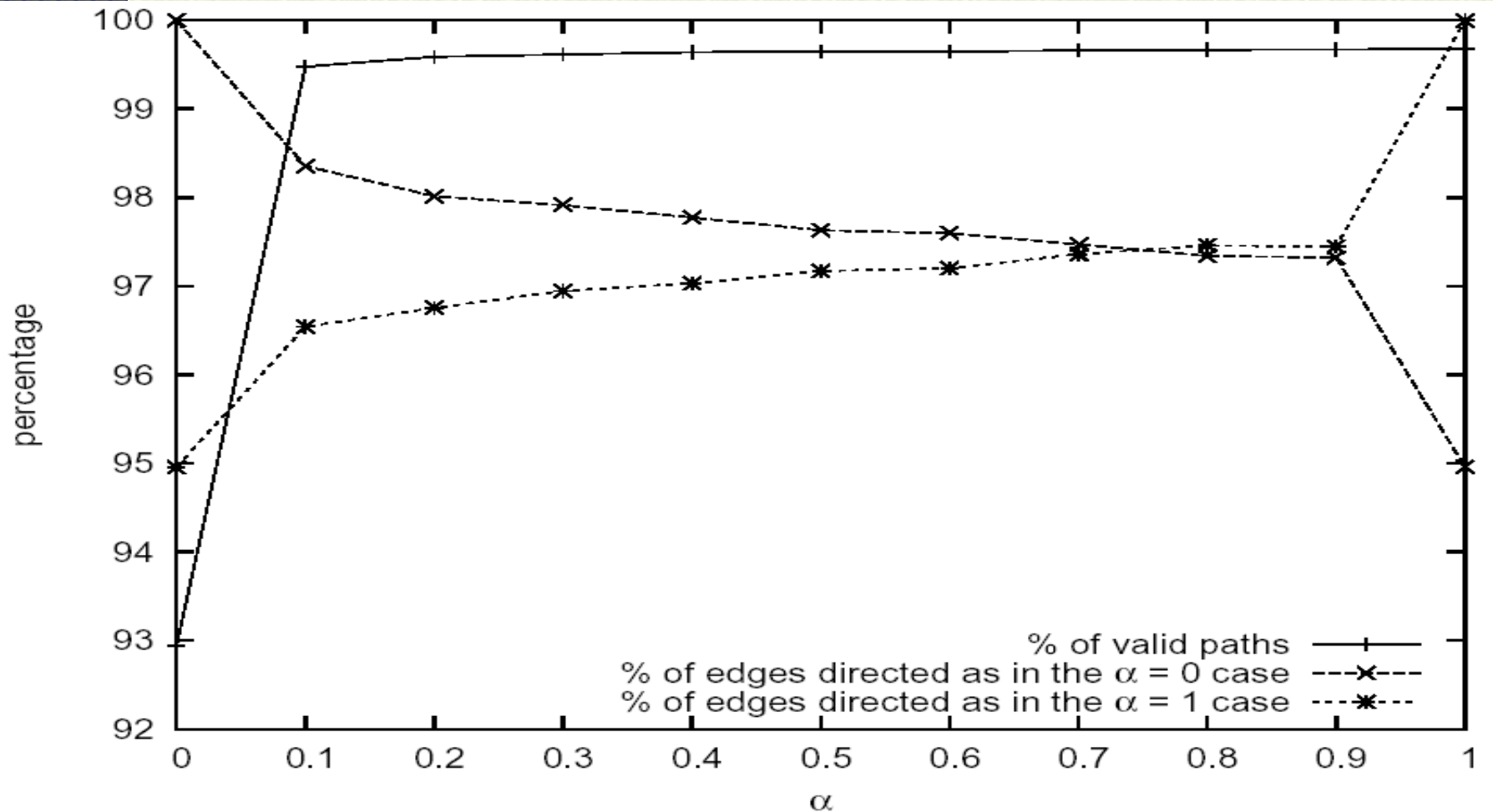
$$f(d_i^-, d_i^+) = \frac{d_i^+ - d_i^-}{d_i^+ + d_i^-} \log(d_i^+ + d_i^-).$$

# Execution process

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- # BGP data: RouteViews and 18 route servers from traceroute.org (1025775 paths, 17557 ASs, 37021 1-link clauses/edges, 382917 2-link clauses/pairs of adjacent edges)
- # Resolve siblings that can be found via WHOIS
- # Direct all edges along the node degree gradient
- # Strip non-conflict edges whose non-conflict direction, inferred by topological sorting in  $G_{2SAT}$ , satisfy all clauses containing corresponding variables (remains: 1590 ASs, 4249 1-links, 23460 2-links)
- # Execute the algorithm for different values of  $\alpha$ : intensive usage of:
  - LEDA library
  - DSDP solver

# First results: invalid paths



# First results: AS hierarchy

Table 2: Ranking of ASs induced by our inference algorithm. The ranks of the top five ASs for  $\alpha = 0$  and  $\alpha = 1$  are shown for different values of  $\alpha$ . The AS numbers are matched to an AS name using the WHOIS databases.

AS #	AS name	AS outdegree	$\alpha = 0.0$	$\alpha = 0.2$	$\alpha = 0.4$	$\alpha = 0.6$	$\alpha = 0.8$	$\alpha = 1.0$
701	UUNET	2373	0-1	0-173	0-217	1-242	1-252	17-476
1239	Sprint	1787	1-1	0-173	0-217	1-242	1-252	17-476
7018	AT&T	1723	2-1	0-173	0-217	1-242	1-252	17-476
3356	Level 3	1085	3-1	0-173	0-217	1-242	1-252	17-476
209	Qwest	1072	4-1	0-173	0-217	1-242	1-252	17-476
11551	Pressroom Services	2	1742-941	1419-398	1435-391	1449-390	1457-386	0-4
6721	Nextra Czech Net	3	1742-941	833-88	853-90	874-90	884-89	0-4
3643	Sprint Australia	17	194-1	222-1	233-1	261-1	268-1	0-4
1243	Army Info. Systems	2	2683-62853	2753-14655	1435-391	1449-390	1457-386	0-4
6712	France Transpac	2	2683-62853	2753-14655	2774-14634	298-2	1-252	4-13

# Conclusion and future work

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## # What's done

- Using a standard multiobjective optimization method, resolved issues associated with two polar approaches

## # What's to be done

- Peering
- Further analysis of the 'trade-off surface'
- Further analysis of the directed AS graph
- Validation
- Other AS-ranking mechanisms
- AS-ranking webpage update

[http://www.caida.org/analysis/topology/rank\\_as/](http://www.caida.org/analysis/topology/rank_as/)

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