

# Identifying Influential Spreaders in Complex Networks

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<http://arxiv.org/abs/1001.5285>

**Q1:** Definition of spreading efficiency?

**Q2:** What determines spreading efficiency?

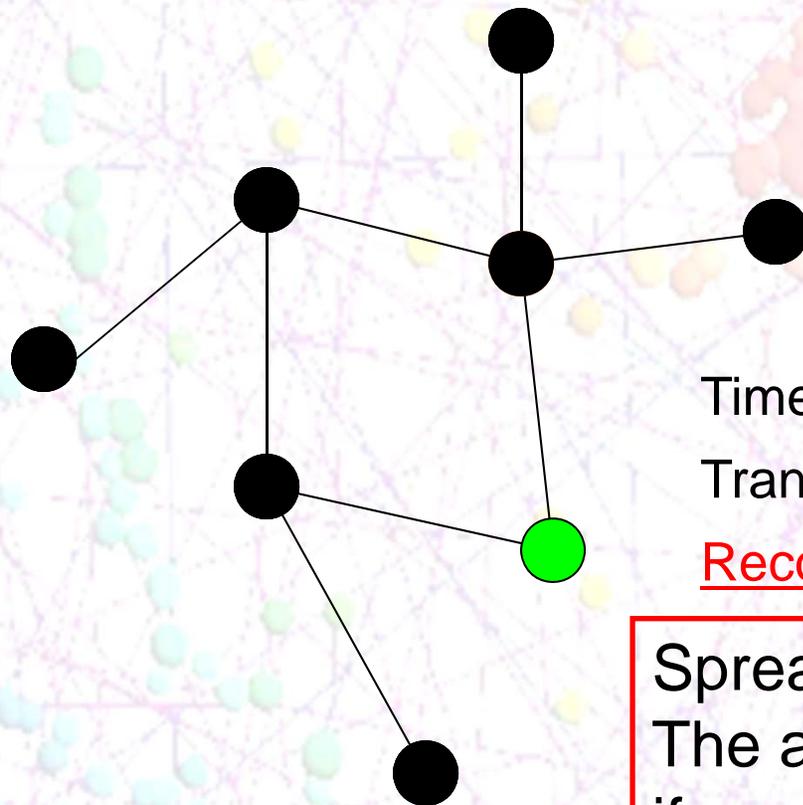
**Q3:** Who are the most efficient spreaders?



# Spreading Processes: Examples and Models

- Examples:
- Infectious Diseases (smallpox, influenza...)
- Innovations, Rumor, Ideas
- Computer Viruses (spreading via email)

## The SIR Model



● “S”usceptible” (unaffected) individual.

● “I”nfected” (affected) individual.

● “R”ecovered” individual.

Time to “recover”  $T_R = 2$

Transmission probability  $\beta = 0.5$

Recovered individuals can not be infected!!!

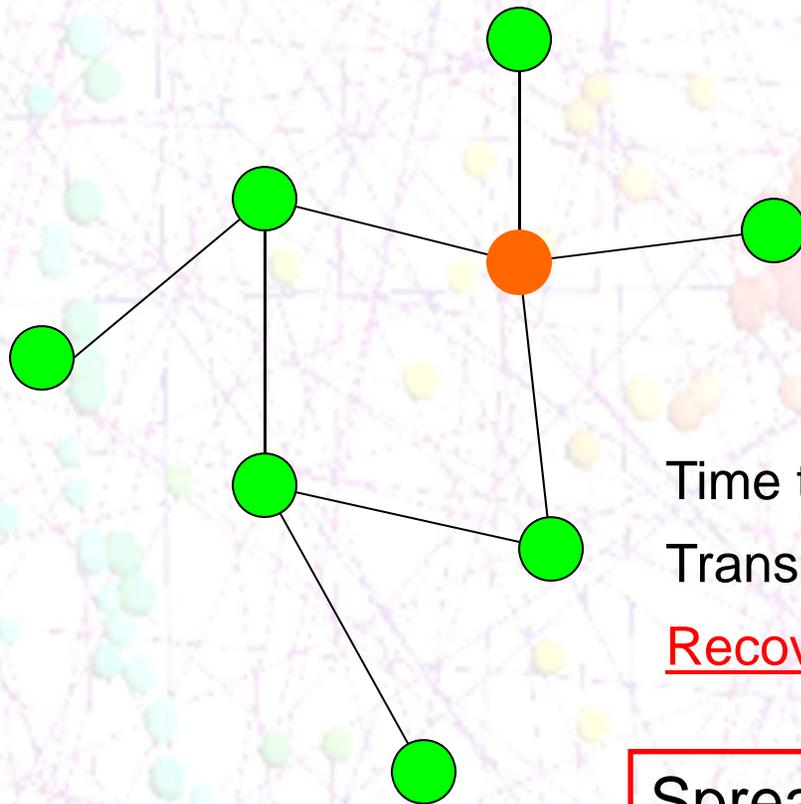
Spreading efficiency:  $\langle M_i \rangle$

The average number of infected nodes  
if spreading starts at node  $i$

# Spreading Processes: Examples and Models

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## The SIS Model



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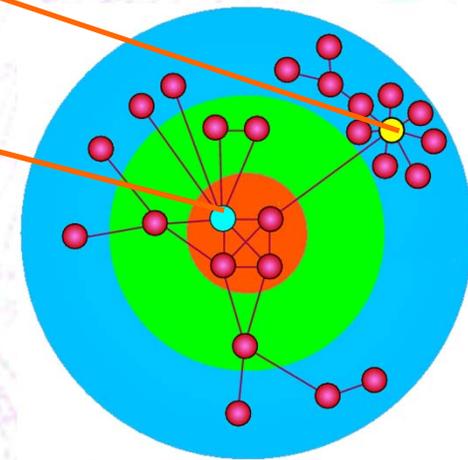
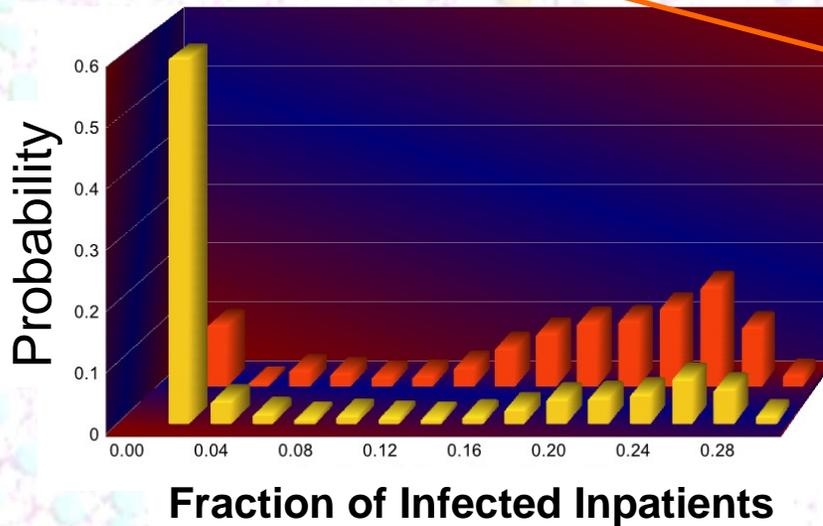
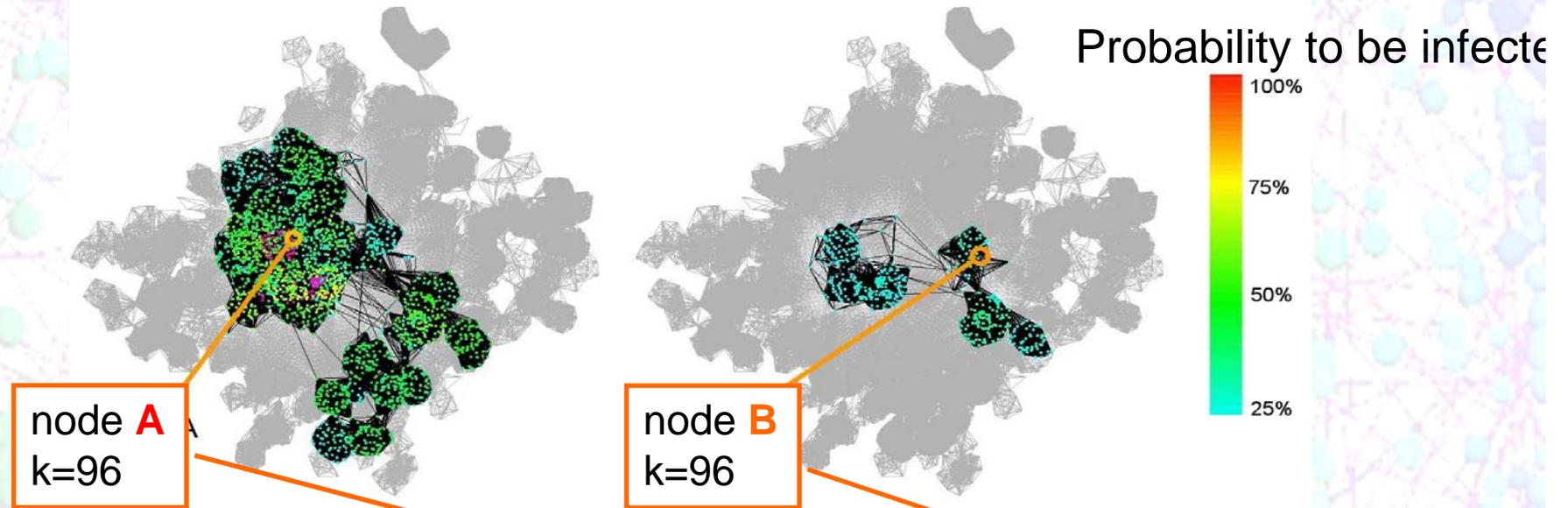
Transmission probability  $\beta = 0.5$

Recovered individuals can be infected again!!!

Spreading efficiency:  $\rho_i(t)$   
Probability node  $i$  is infected at time  $t$

# Spreading efficiently determined by node placement!

Hospital Network: Inpatients in the same quarters connected with links

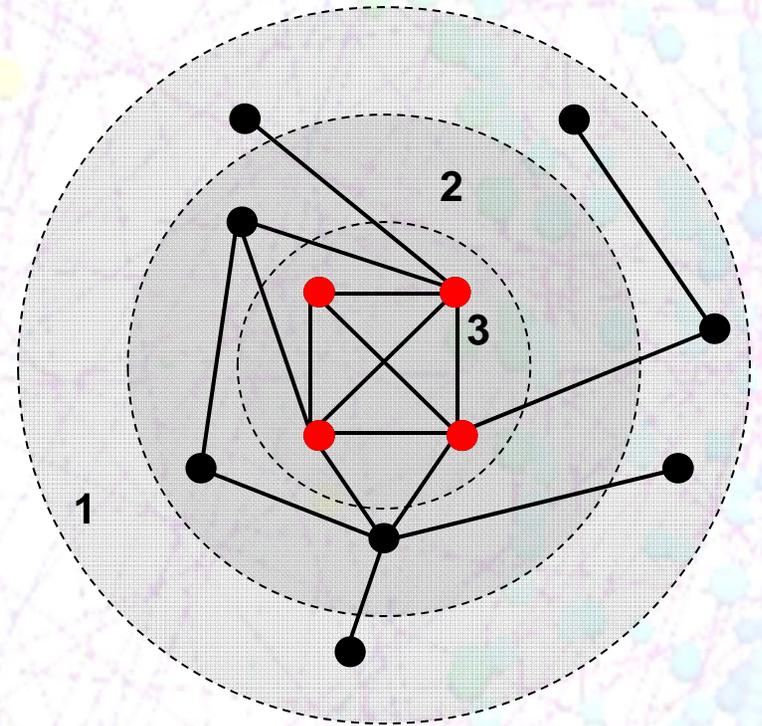


# *k*-cores and *k*-shells determine node placement

**K-core: sub-graph with nodes of degree at least  $k$  inside the sub-graph.**

## Pruning Rule:

- 1) Remove all nodes with  $k=1$ .  
Some remaining nodes may now have  $k = 1$ .
- 2) Repeat until there is no nodes with  $k = 1$ .
- 3) The remaining network forms the 2-core.
- 4) Repeat the process for higher  $k$  to extract other cores



S. B. Seidman, Social Networks, **5**, 269 (1983).

**K-shell is a set of nodes that belongs to the K-core  
but NOT to the K+1 core**

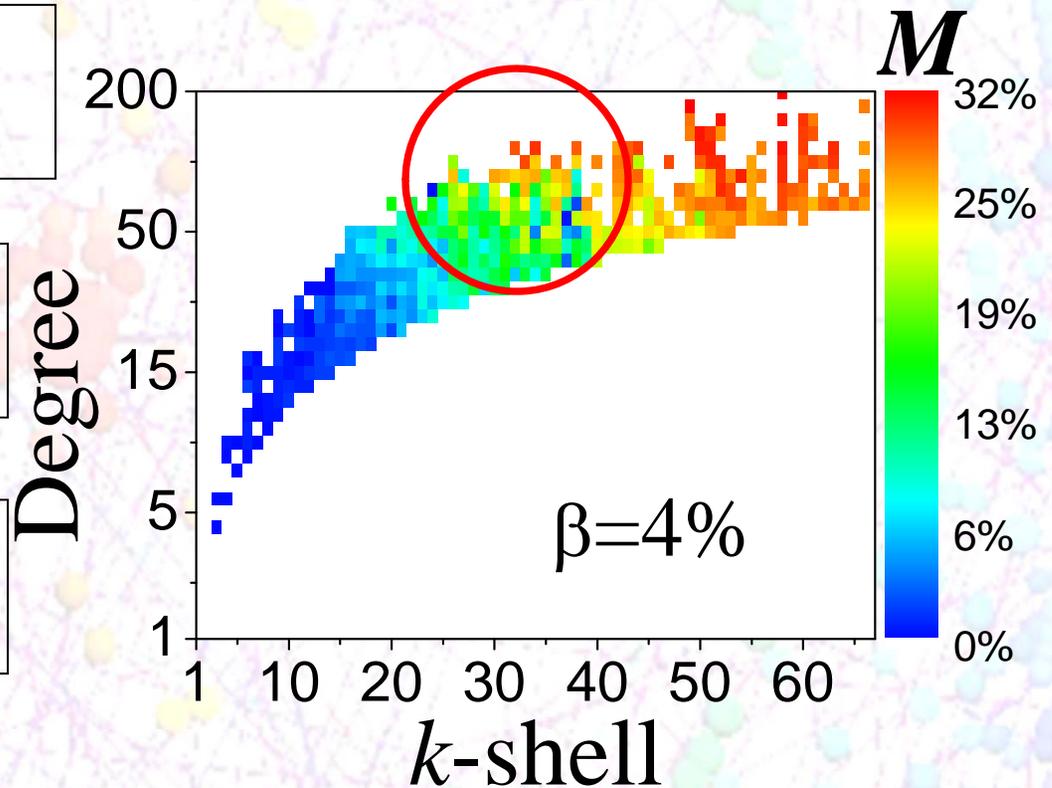
## Identifying efficient spreaders in the hospital network (SIR)

- (1) For every individual  $i$  measure the average fraction of individuals  $M_i$  he or she would infect (spreading efficiency).
- (2) Group individuals based on the number of connections and the  $k$ -shell value.

**A.** Most efficient spreaders occupy high  $k$ -shells.

**B.** For fixed  $k$ -shell  $\langle M \rangle$  is independent of  $k$ .

**C.** A lot of hubs are inefficient spreaders.



Three candidates:  
Degree,  $k$ -shell, betweenness centrality

## Imprecision functions test the merits of degree, k-shell and centrality

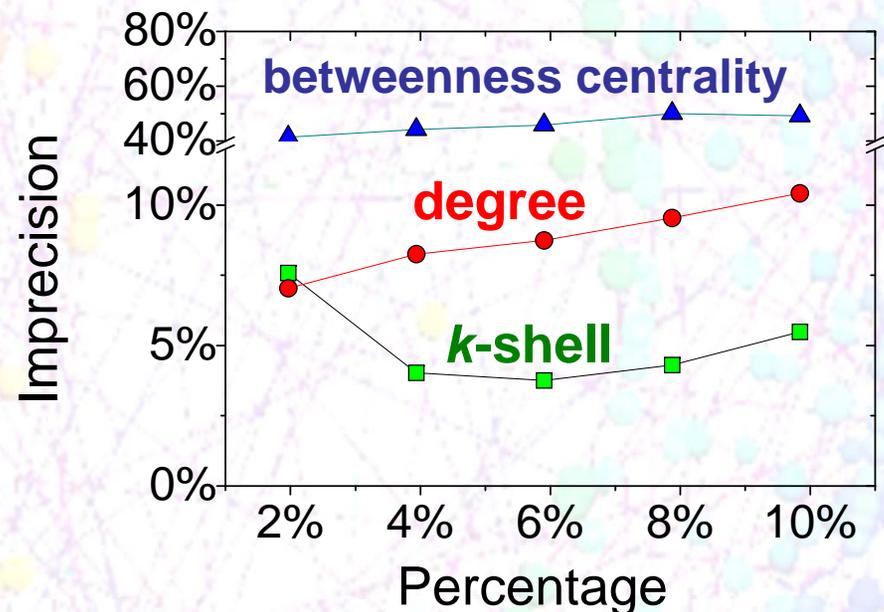
For given percentage  $p$

- Find  $Np$  the most efficient spreaders (as measured by  $M$ )
- Calculate the average infected mass  $M_{EFF}$ .
- Find  $Np$  the nodes with highest *k-shell* indices.
- Calculate the average infected mass  $M_{kshell}$ .

Imprecision function:

$$\varepsilon(p) = 1 - \frac{M_{kshell}(p)}{M_{EFF}(p)}$$

Measure the imprecision for K-shell, degree and centrality.

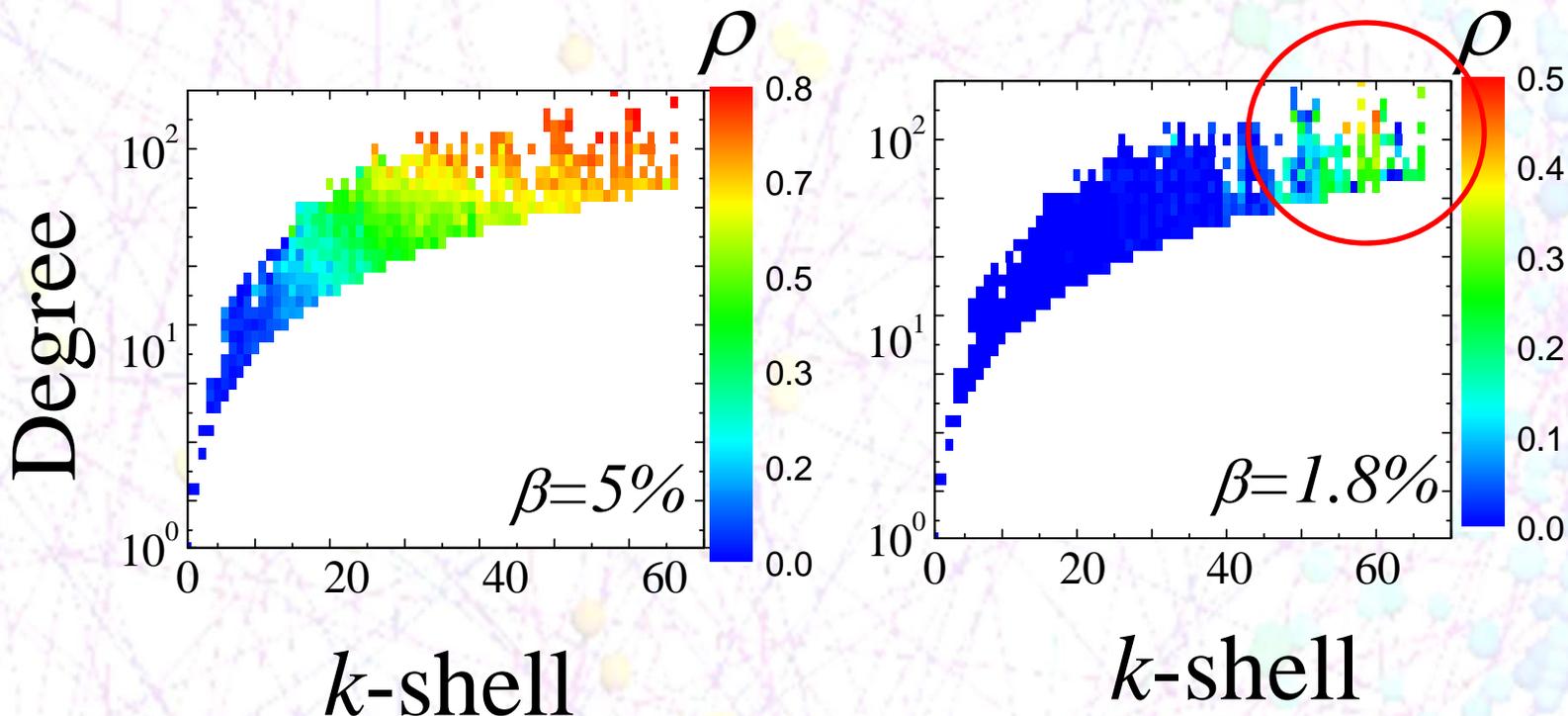


*k-shell* is the most robust spreading efficiency indicator.  
(followed by degree and betweenness centrality)

## Identifying efficient spreaders in the hospital network (SIS)

SIS: Number of infected nodes reaches endemic state (equilibrium)

Persistence  $\rho_i(t)$  (probability node  $i$  is infected at time  $t$ )



High  $k$ -shells form a reservoir where virus can exist locally.

Consistent with core groups (H. Hethcote et al 1984)

# Summary

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## SIR

1)  $k$ -shell value is a reliable indicator of spreading efficiency. The most efficient spreaders occupy the innermost  $k$ -shells.

2) Multiple source spreading is enhanced when one “repels” sources. (*Discussed in the paper*)

## SIS

3) High  $k$ -shells form a reservoir where virus can survive locally and infect neighbor nodes.

4) High  $k$ -shells may decrease epidemic threshold.

5) Immunization/Removal of high  $k$ -shells helps to suppress virus persistence.