

Identifying Influential Spreaders in Complex Networks

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<http://arxiv.org/abs/1001.5285>

Q1: Definition of spreading efficiency?

Q2: What determines spreading efficiency?

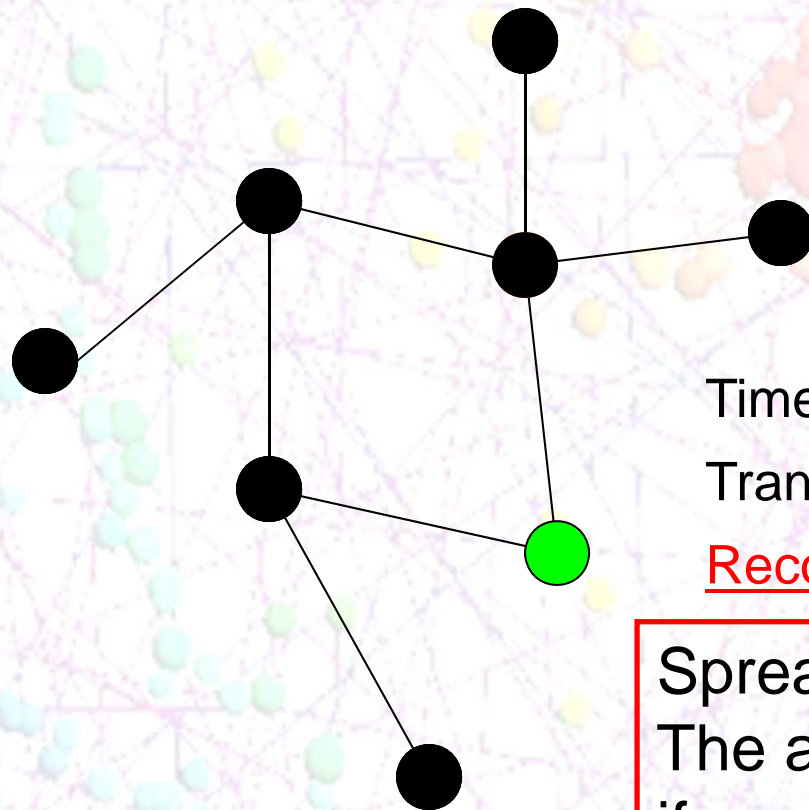
Q3: Who are the most efficient spreaders?



Spreading Processes: Examples and Models

- Examples:
- Infectious Diseases (smallpox, influenza...)
- Innovations, Rumor, Ideas
- Computer Viruses (spreading via email)

The SIR Model



● “S”usceptible” (unaffected) individual.

● “I”nfected” (affected) individual.

● “R”ecovered” individual.

Time to “recover” $T_R = 2$

Transmission probability $\beta = 0.5$

Recovered individuals can not be infected!!!

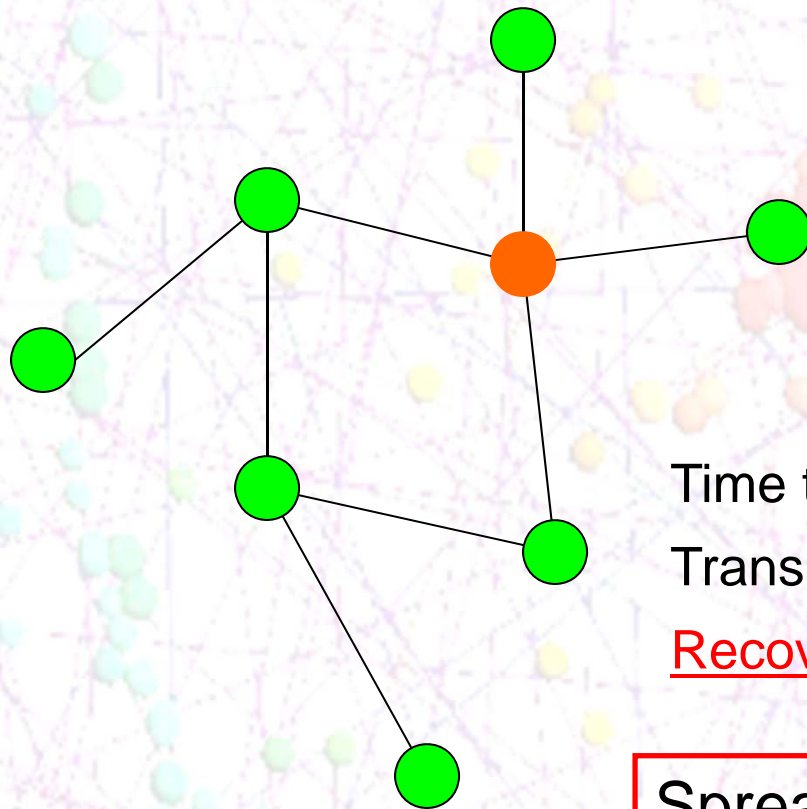
Spreading efficiency: $\langle M_i \rangle$

The average number of infected nodes
if spreading starts at node i

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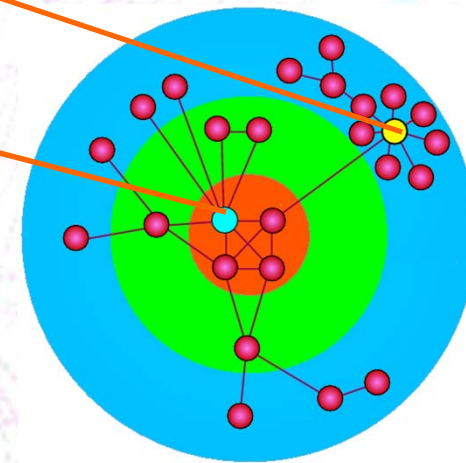
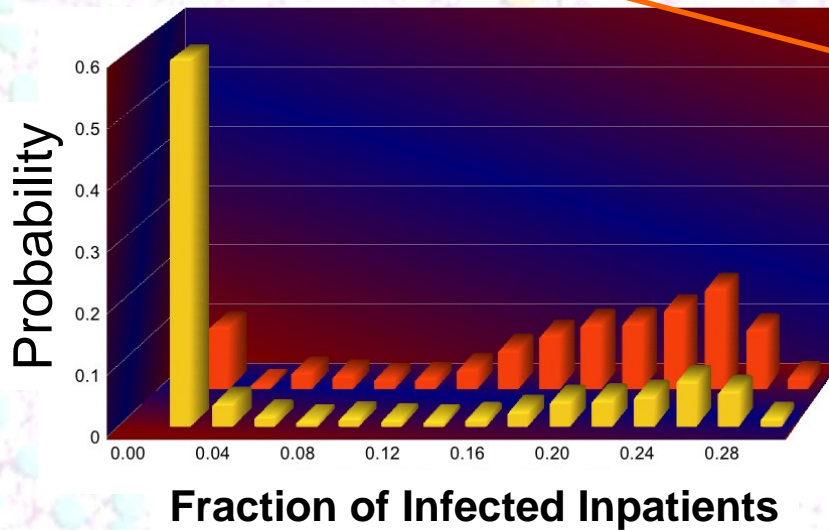
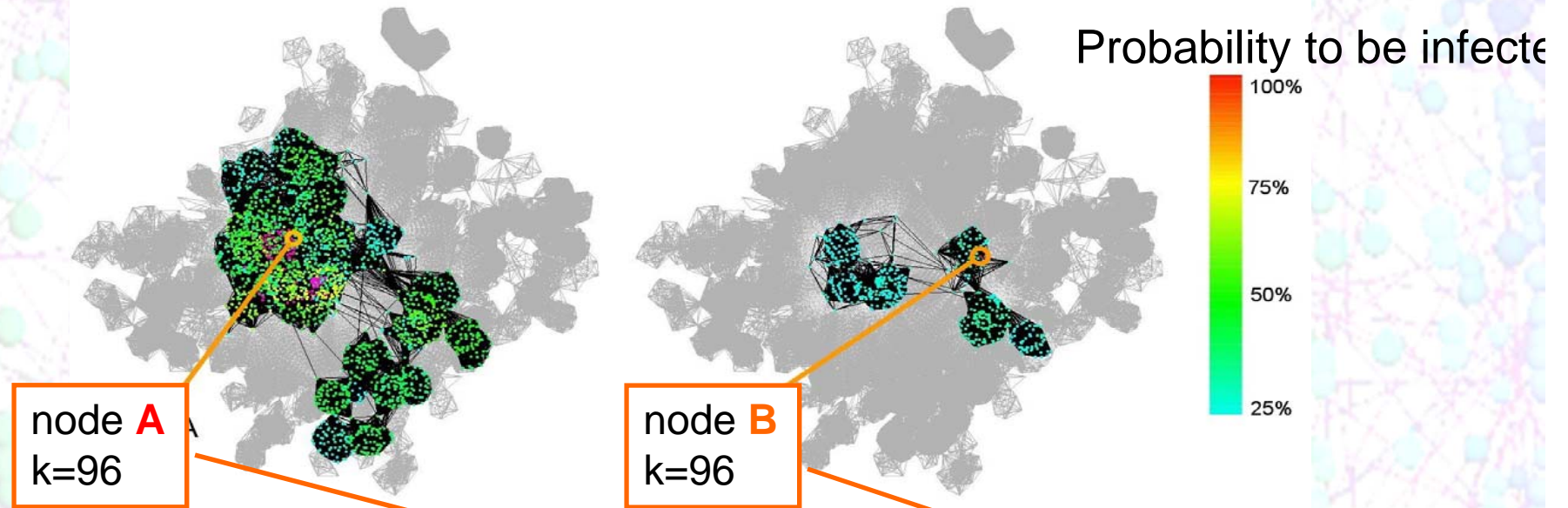
Transmission probability $\beta = 0.5$

Recovered individuals can be infected again!!!

Spreading efficiency: $\rho_i(t)$
Probability node i is infected at time t

Spreading efficiently determined by node placement!

Hospital Network: Inpatients in the same quarters connected with links

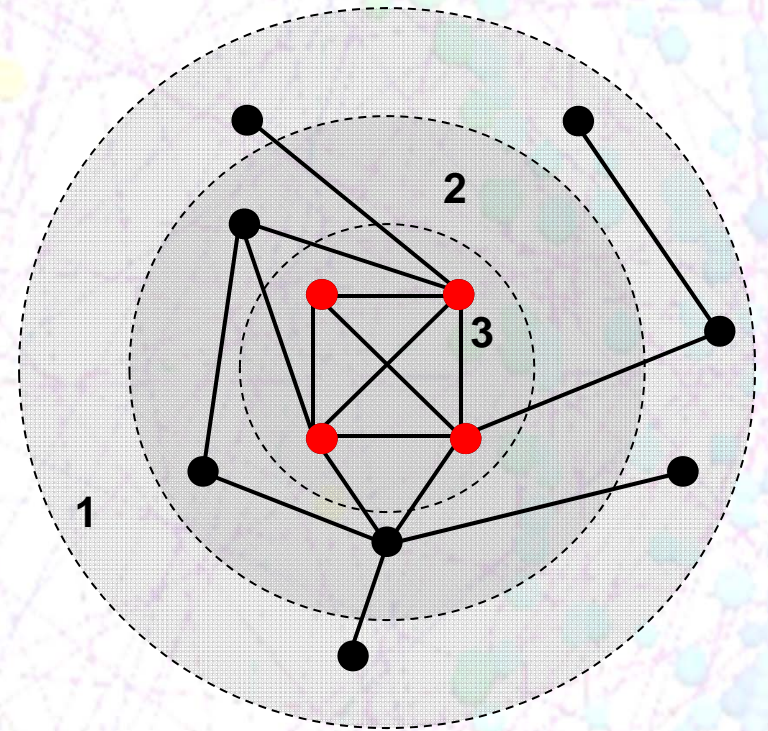


k-cores and *k*-shells determine node placement

K-core: sub-graph with nodes of degree at least *k* inside the sub-graph.

Pruning Rule:

- 1) Remove all nodes with $k=1$.
Some remaining nodes may now have $k = 1$.
- 2) Repeat until there is no nodes with $k = 1$.
- 3) The remaining network forms the 2-core.
- 4) Repeat the process for higher k to extract other cores



S. B. Seidman, Social Networks, **5**, 269 (1983).

**K-shell is a set of nodes that belongs to the K-core
but NOT to the K+1 core**

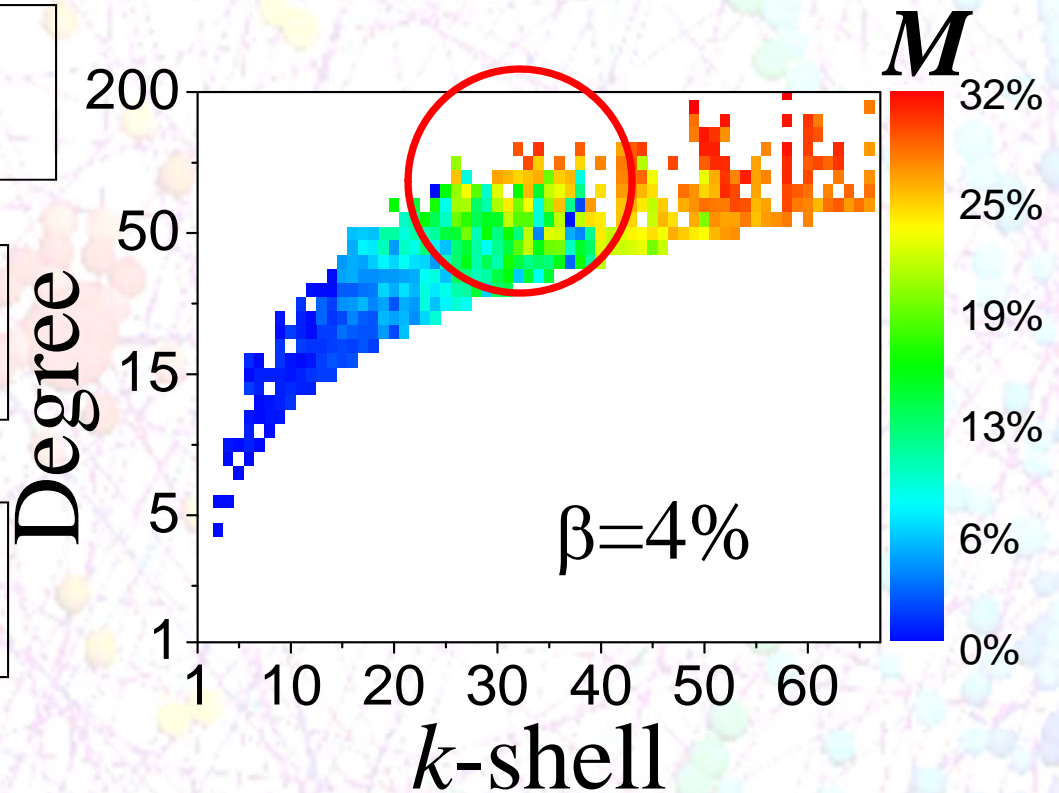
Identifying efficient spreaders in the hospital network (SIR)

- (1) For every individual i measure the average fraction of individuals M_i he or she would infect (spreading efficiency).
- (2) Group individuals based on the number of connections and the k -shell value.

A. Most efficient spreaders occupy high k -shells.

B. For fixed k -shell $\langle M \rangle$ is independent of k .

C. A lot of hubs are inefficient spreaders.



Three candidates:
Degree, k -shell, betweenness centrality

Imprecision functions test the merits of degree, k-shell and centrality

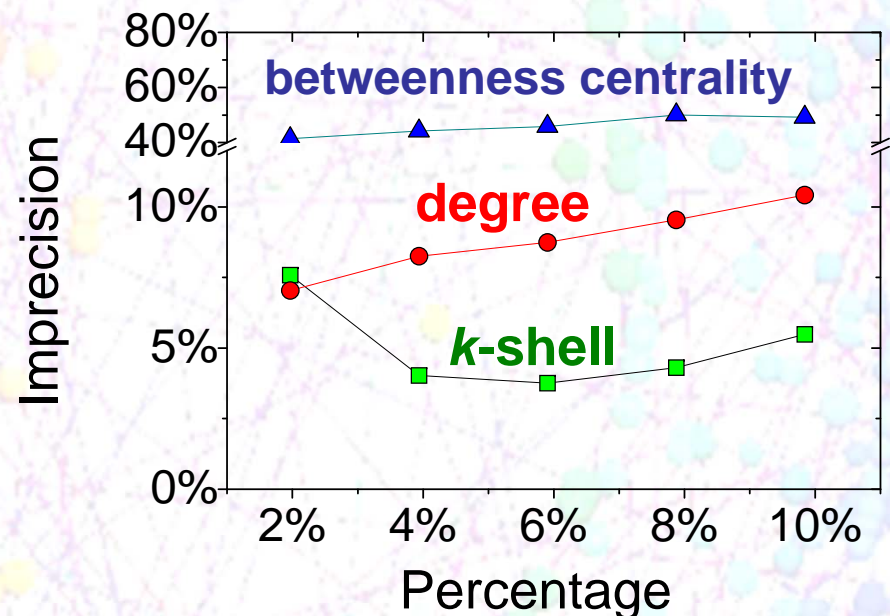
For given percentage p

- Find Np the most efficient spreaders (as measured by M)
- Calculate the average infected mass M_{EFF} .
- Find Np the nodes with highest *k-shell* indices.
- Calculate the average infected mass M_{kshell} .

Imprecision function:

$$\varepsilon(p) = 1 - \frac{M_{kshell}(p)}{M_{EFF}(p)}$$

Measure the imprecision for K-shell, degree and centrality.

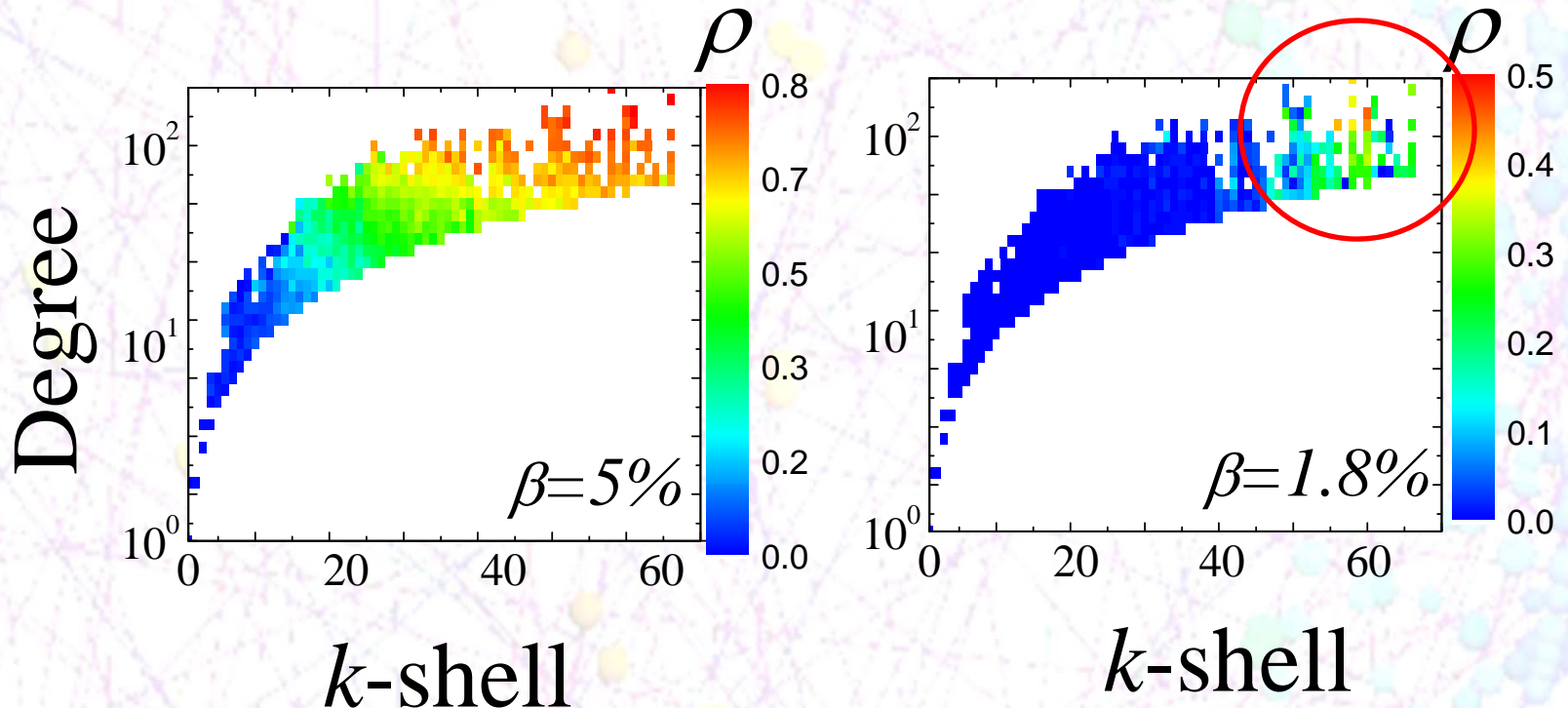


k-shell is the most robust spreading efficiency indicator.
(followed by degree and betweenness centrality)

Identifying efficient spreaders in the hospital network (SIS)

SIS: Number of infected nodes reaches endemic state (equilibrium)

Persistence $\rho_i(t)$ (probability node i is infected at time t)



High k -shells form a reservoir where virus can exist locally.

Consistent with core groups (H. Hethcote et al 1984)

Summary

SIR

- 1) k -shell value is a reliable indicator of spreading efficiency. The most efficient spreaders occupy the innermost k -shells.
- 2) Multiple source spreading is enhanced when one “repels” sources. (*Discussed in the paper*)

SIS

- 3) High k -shells form a reservoir where virus can survive locally and infect neighbor nodes.
- 4) High k -shells may decrease epidemic threshold.
- 5) Immunization/Removal of high k -shells helps to suppress virus persistence.