Navigating with Power Laws

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Milgram Study (1967)

A question of social connectedness

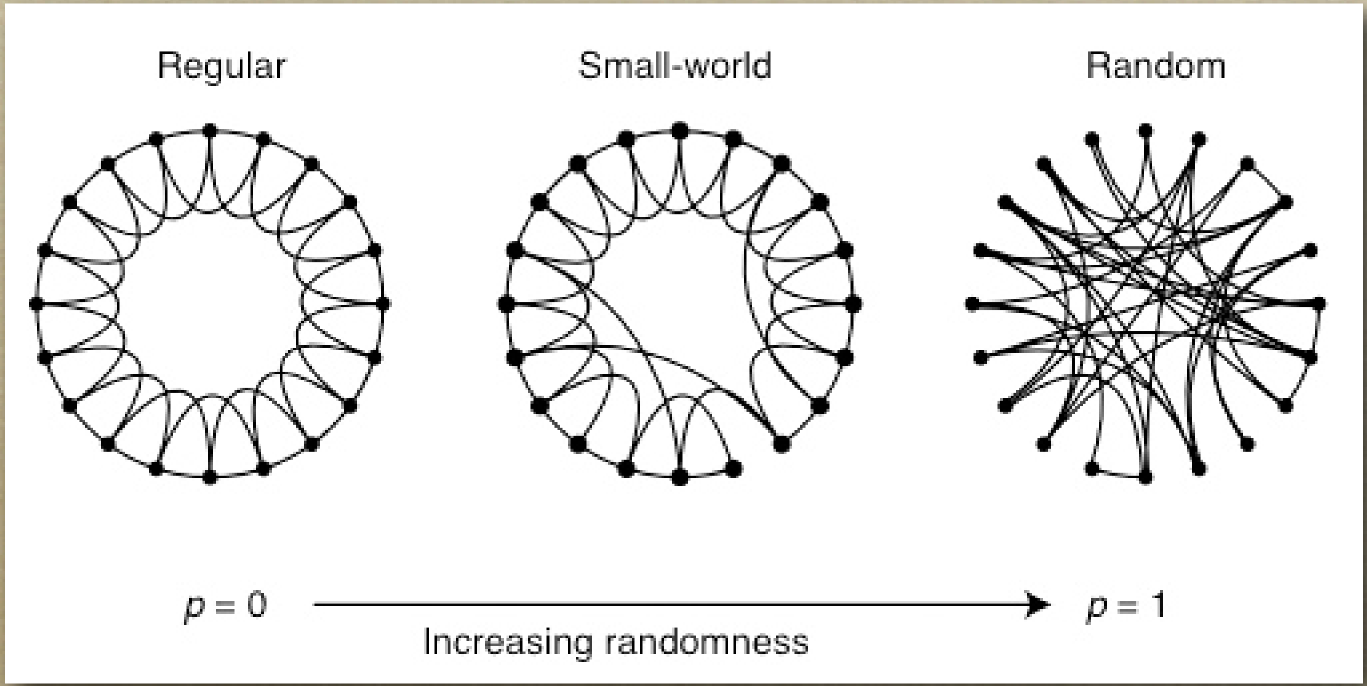
- *60 letters sent to Wichita, Kansas*
- *Destination: wife of divinity stud., Cambridge, Ma.*
- *Only 3 arrived*
- *Subsequent studies: mean path length ~6*

Discoveries

- *Surprisingly short paths; "small world" phenom.*
- *Shorts paths are locally discoverable*

S. Milgram, "The small world problem." *Psychology Today* **2** (1967) 60--67.

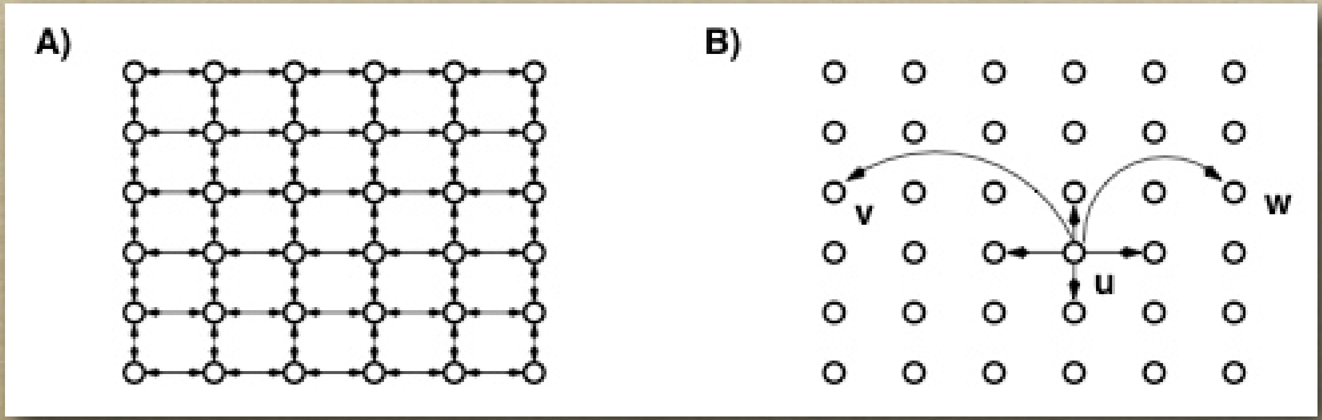
Watts-Strogatz Model (1998)



- *Modeled existence of short paths only*
- *diameter* $\log(n)$

D.J. Watts and S.H. Strogatz, “Collective dynamics of small-world networks.” *Nature* **393** (1998) 440-442.

Kleinberg Model (2000)



- *Model of navigability/search*
- *Lattice + long range links*
- *(Manhattan) distance metric* $d(u, v) = |u - v|$
- *Local (greedy) navigation in ~* $\log^2(n)$ *steps*

J. Kleinberg. "The small-world phenomenon: an algorithmic perspective." *Proc. 32nd ACM Symposium on Theory of Computing* (2000) 163--170.

An Aside: Finite Size Effects

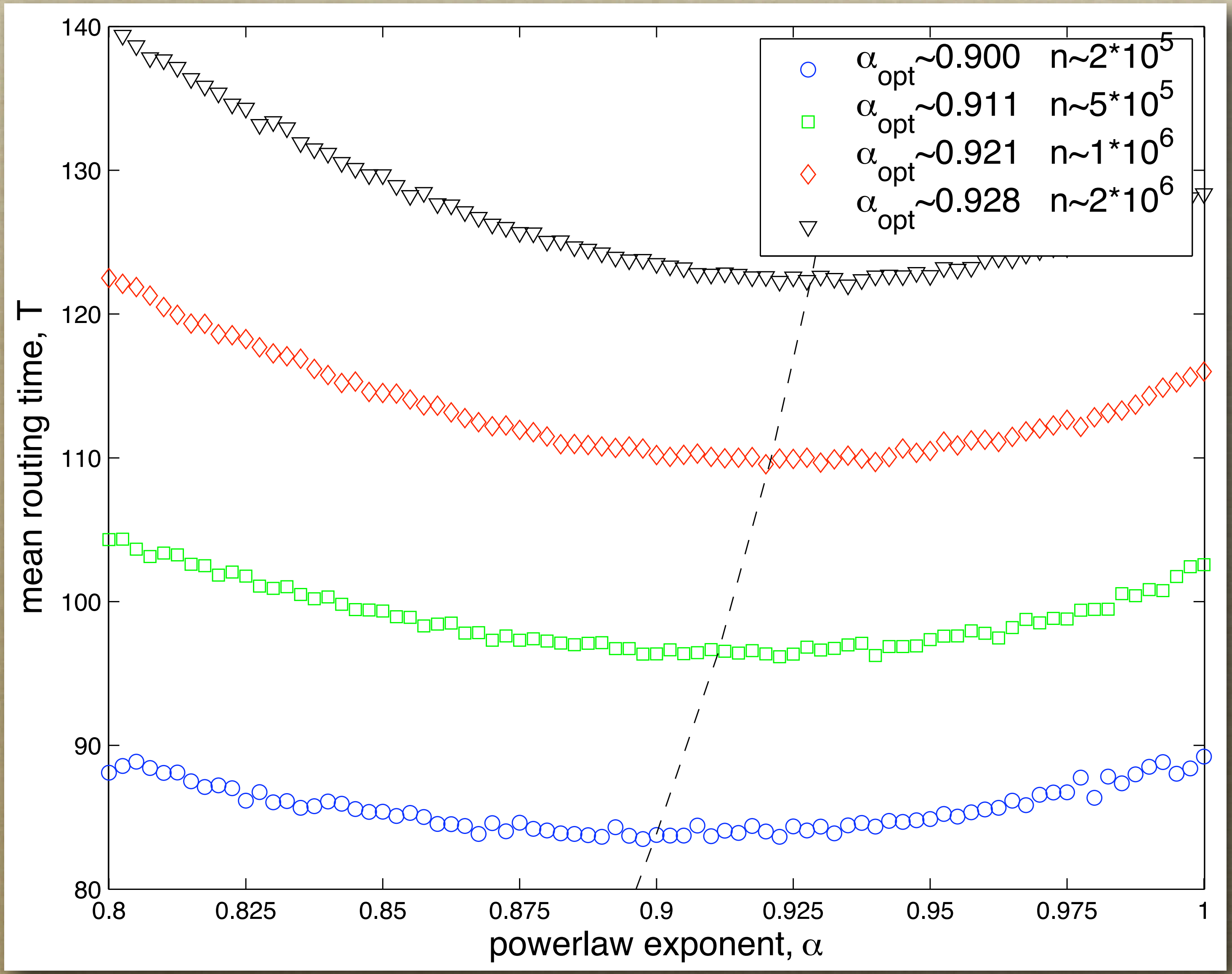
- *Simulate Kleinberg graphs with various* $\alpha \sim d$
- *Measure mean routing time* T
- *Find severe finite size effects* $\alpha_{T_{opt}} \neq d$

  e.g., for $d = 1$

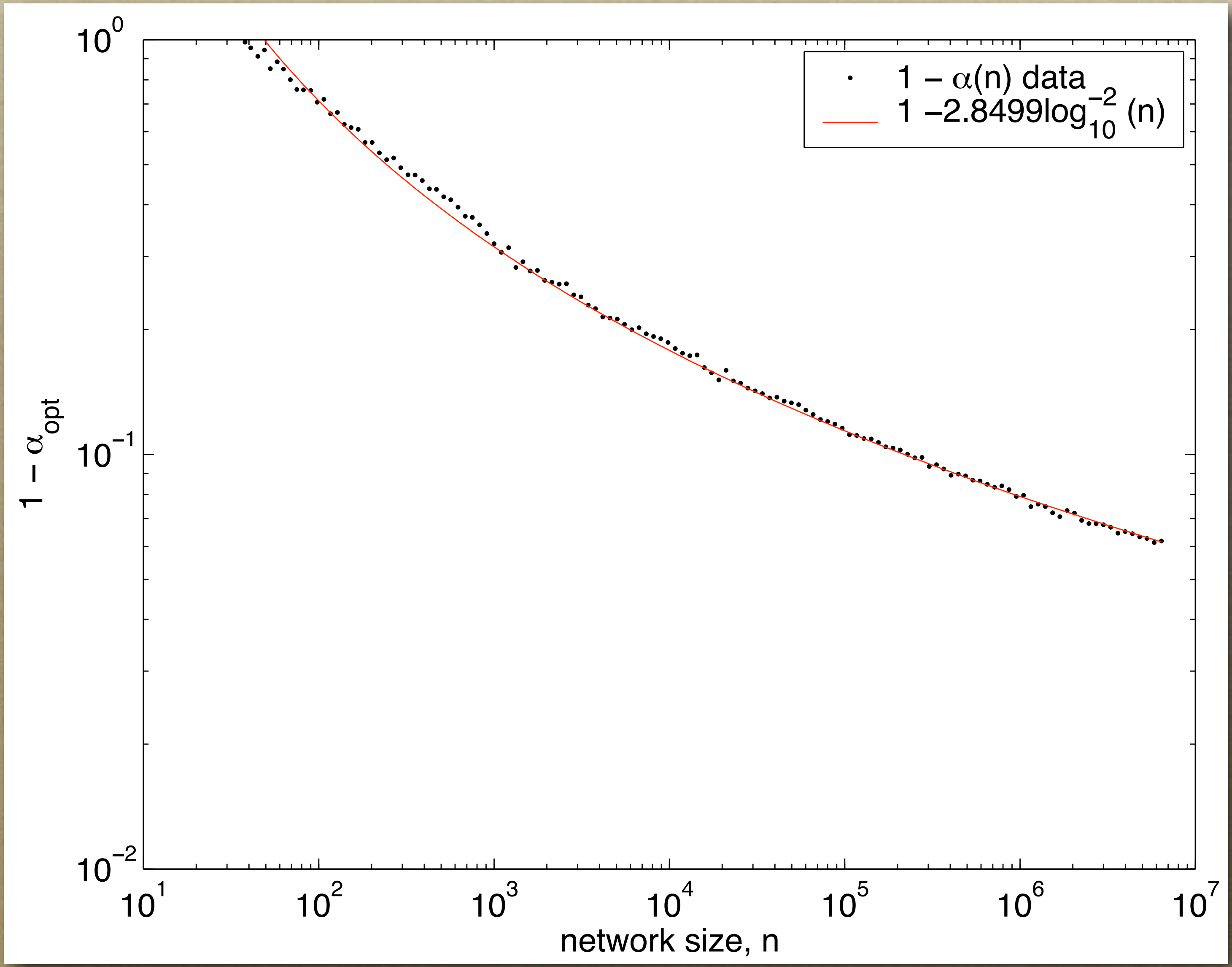
$$\alpha_{T_{opt}}(n) = 1 - \frac{A}{\log^2(n)}$$

- *Keep this in mind for later*

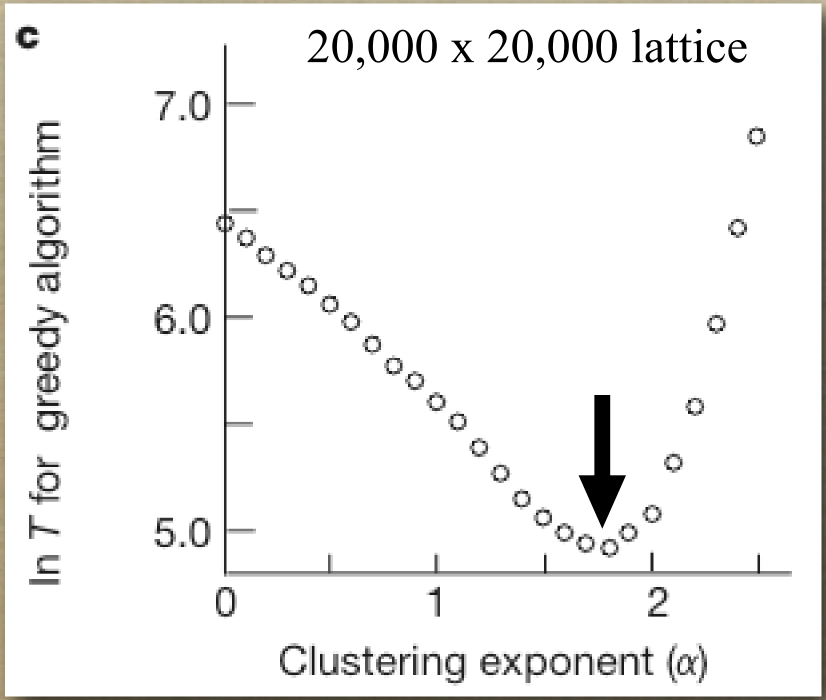
Simulation Results



Convergence



Not a new result



J. Kleinberg, "Navigation in a small world." *Nature* **406** (2000) 845.

Origins of Navigability

Observation

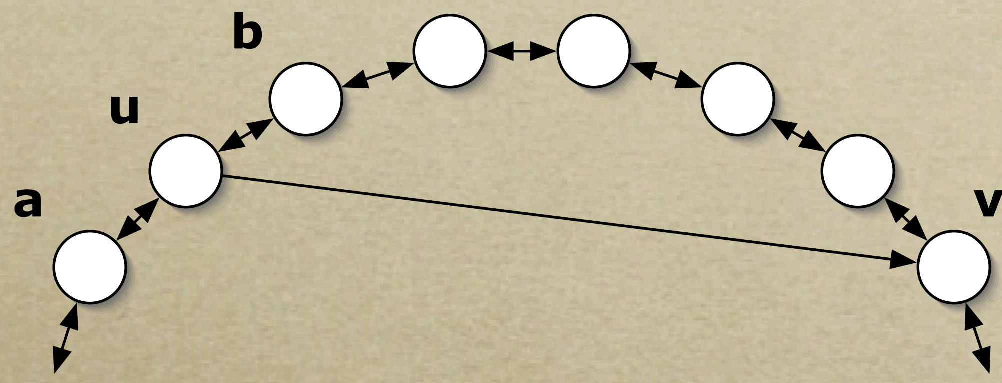
- *Real networks often locally navigable*

  e.g., social network, world wide web

Idea

- *Distributed changes to topology*
- *Greedy changes to improve local navigability*
  - *web surfers on network of home pages*
  - *links changed based on speed of surfing*

Clauset/Moore Model (2003)



- *Same network as Kleinberg*
- *Dynamic, greedy rewiring process*
- *Global attractor for link-length distribution*

$$P(\ell) \to \ell^{-\alpha_{\text{rewired}}} \qquad \alpha_{\text{rewired}} \sim d$$

- *Gives routing time* $T \sim [\log(n)]^2$

Dynamics

1. *choose random pair* (x, y) *(for* $d = 1$*)*
2. *choose random* tolerance T_t *on* $[1, d(x, y)]$
3. *if routing time* $T_r \geq T_t$*, become* frustrated
4. *if* frustrated, *change random long-range link to have length* T_t

Simulation Results

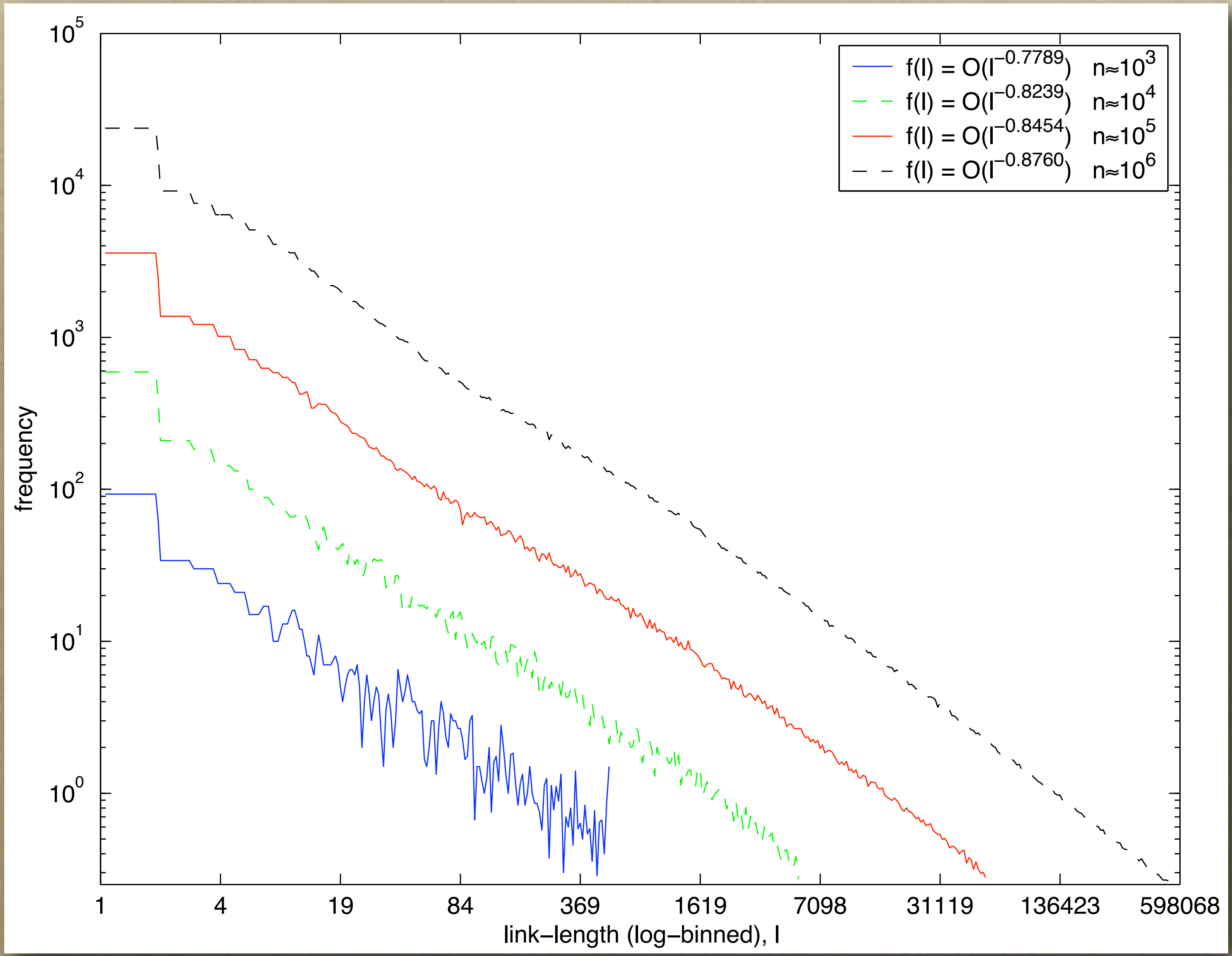
Initial Conditions

All self-loops, i.e., $P(\ell) \sim \ell^{-\infty}$

Stop Criteria

When link-length distribution stabilizes

Rewired Link-Lengths

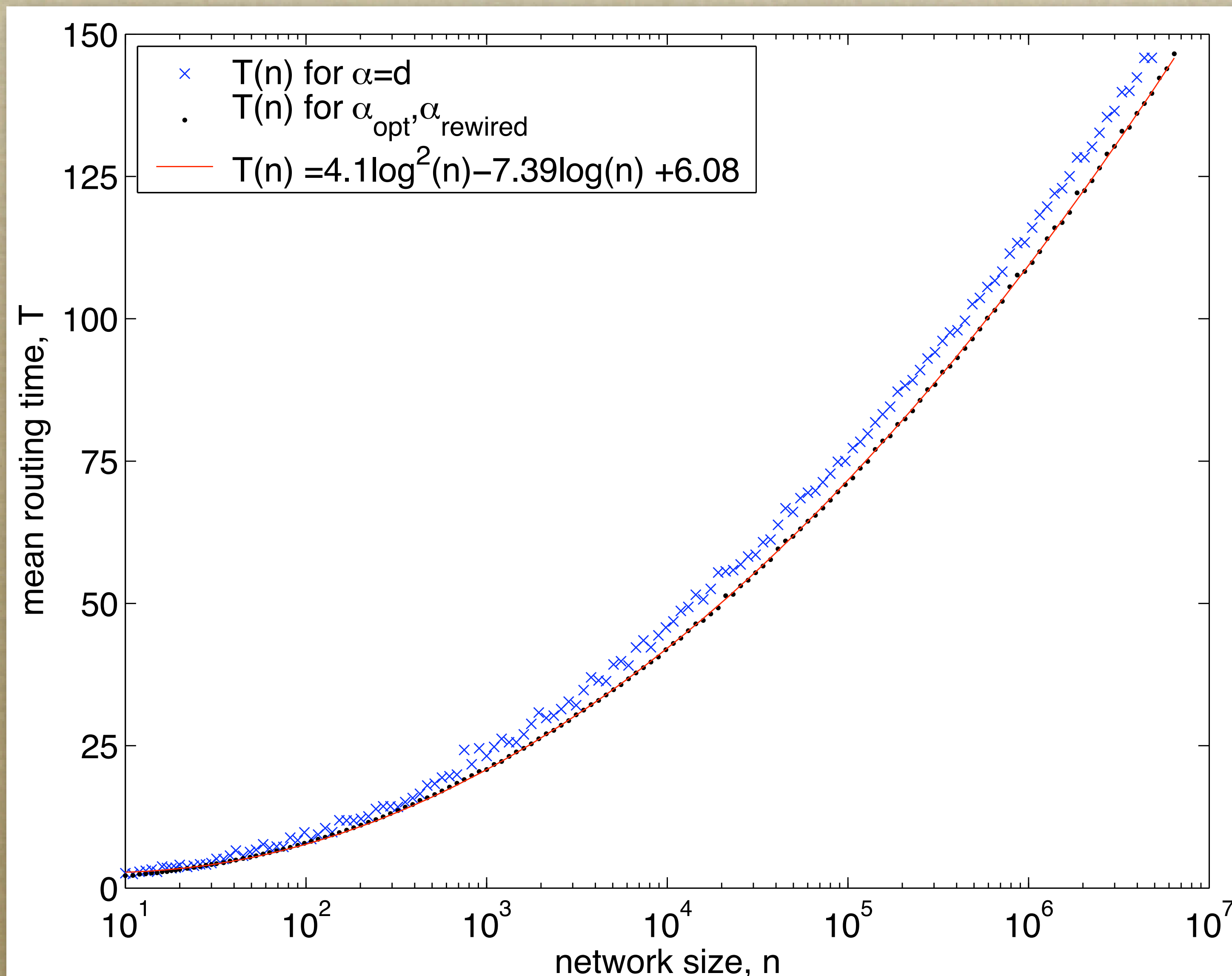


Routing Times

- *Measure mean routing times after stabilization*

- *Fast routing times* $T \sim [\log(n)]^{\alpha_{T_{opt}}}$
- *Recall that* $\alpha_{T_{opt}} \sim d$ *(finite size effects)*

Routing Times



How long until navigable?

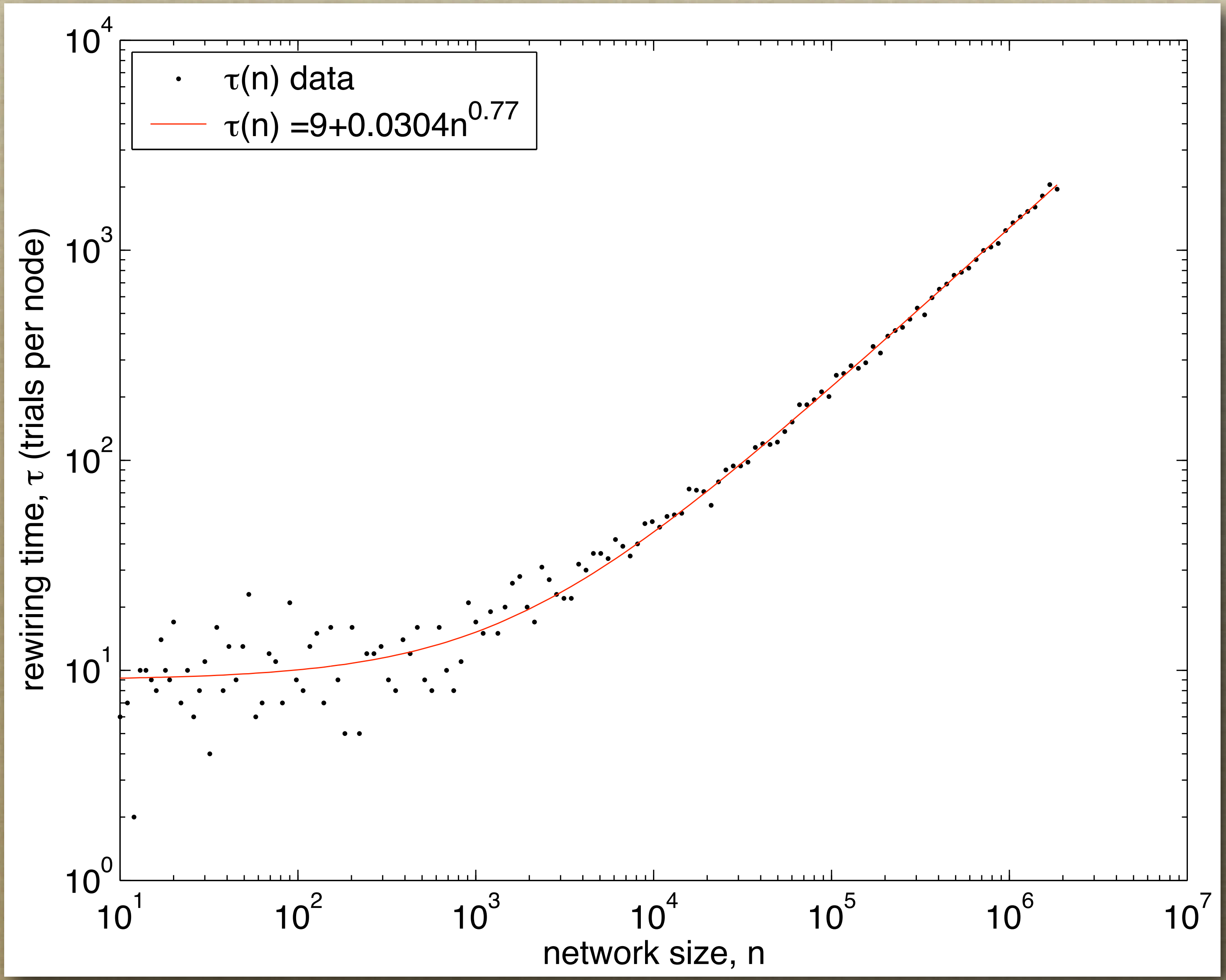
- *Let* rewiring time $\tau(n)$ *be rewiring trials until*

$$T_{\text{rewired}} \leq 1.01 \cdot T_{opt}$$

- $\tau(n)$ *grows as a low-order polynomial*

$$\tau(n) \sim n^{1.77}$$

Time to Navigability



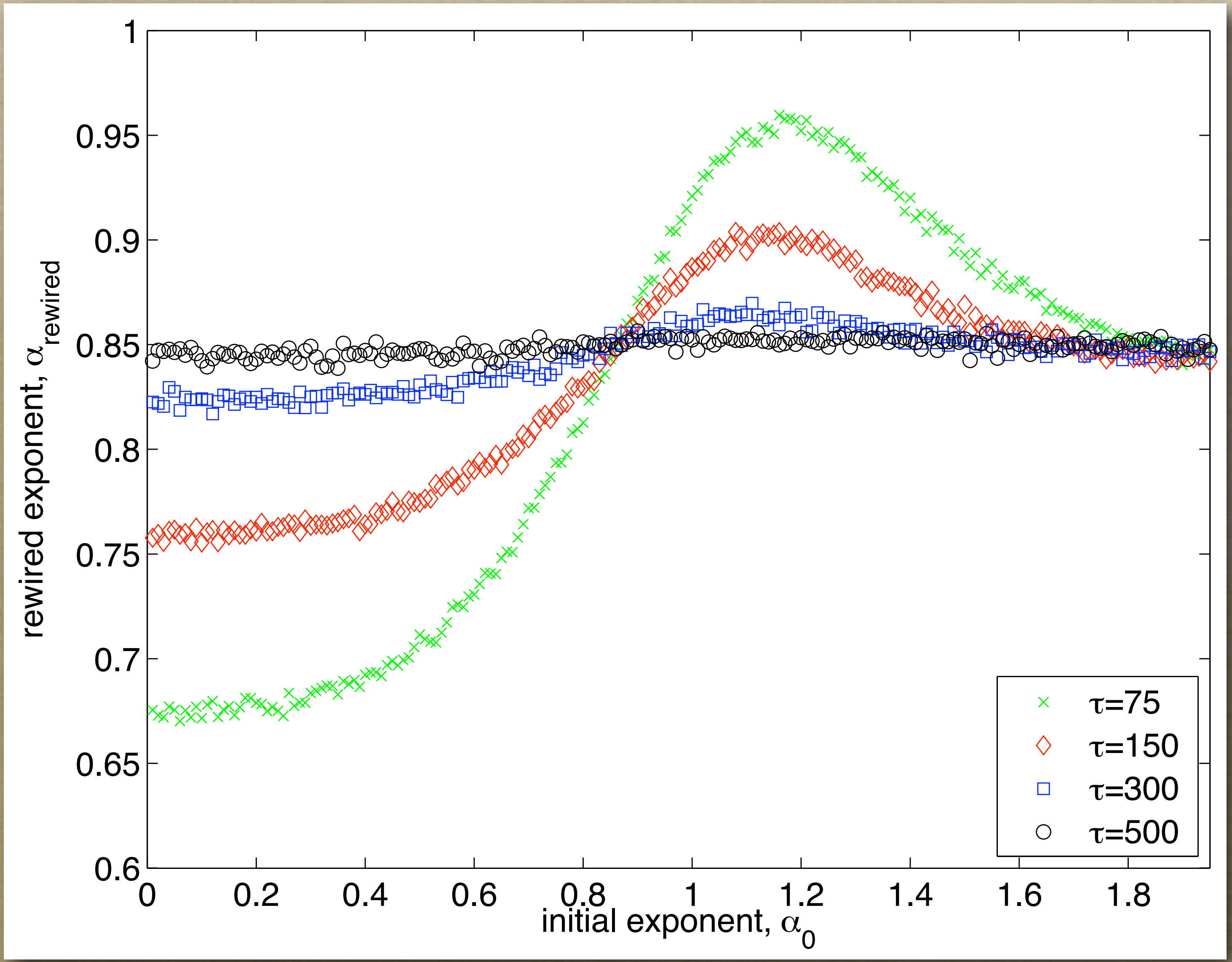
Global Attractor

Initial Condition

- *Link-length distribution* $P(\ell) \sim \ell^{-\alpha_0}$
- *Measure* $\alpha_{\mathrm{rewired}}$ *as function of* α_0

- *Rewired distribution (eventually) independent of initial condition*

Independence



Analytics

- *Distribution of tolerances* T_t

$$P(T_t) = \frac{\log n - \log T_t}{n - 1 - \log n}$$

- *Otherwise, we don't know much*

- *If* $\alpha = d$*, then* $E[\text{new length}] = E[\text{old length}]$
- *What is* $P(\text{frustrated} \mid d(u, v))$ *?*

Thoughts

- *Navigability can come from distributed behavior*
- *Natural/intuitive mechanism*
- *Process is adaptive to changes in size, etc.*
- *Analytics hard (full-history problems)*
- *Power laws emerge spontaneously - why?*

- *How does destination popularity effect rewired topol.*
- *Preprint at* cond-mat/0309415